RSOME + COPT: Using Exponential Cones in Robust Optimization

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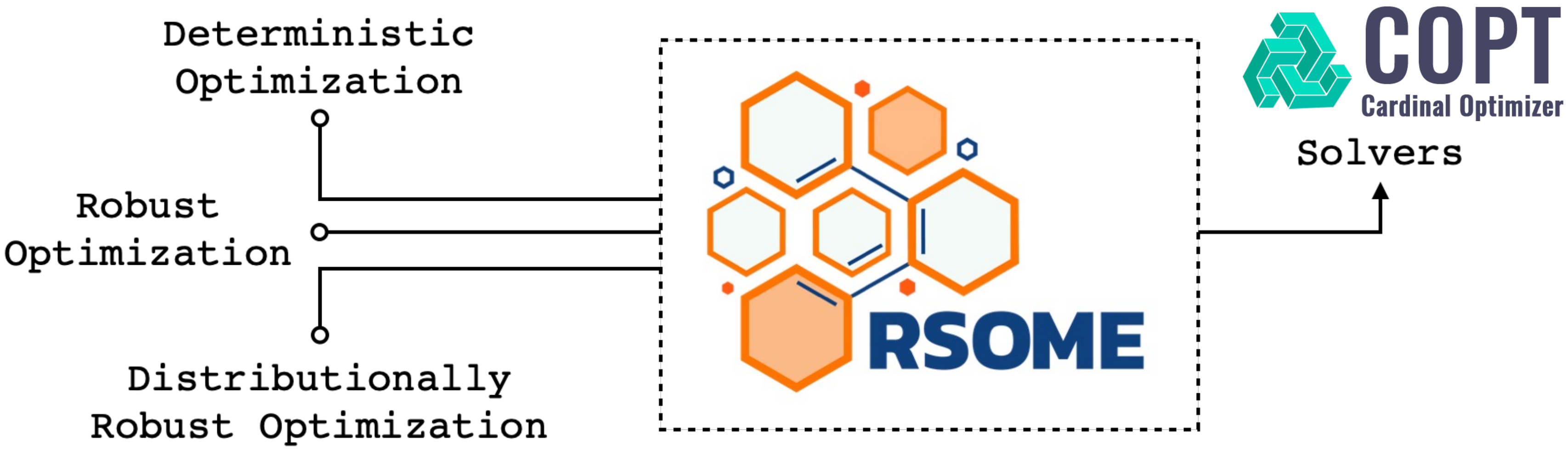
Outline

- Introduction (Part I)
    - RSOME + COPT
    - Exponential cone
    - Optimization under uncertainty
- Example: electric vehicle charging scheduling (Part II)
    - Problem description
    - Stochastic optimization model
    - Exponential conic approximation
    - RSOME + COPT implementation
- General framework: moment-dispersion ambiguity (Part III)

RSOME

- RSOME (Robust Stochastic Optimization Made Easy)



Deterministic Optimization

Robust Optimization

Distributionally Robust Optimization

RSOME

COPT Cardinal Optimizer

Solvers

MODELING POWER

Tailored modeling frameworks for robust and distributionally robust optimization.

READABILITY

Consistent with N-dimensional arrays and vectorized operations of the NumPy package.

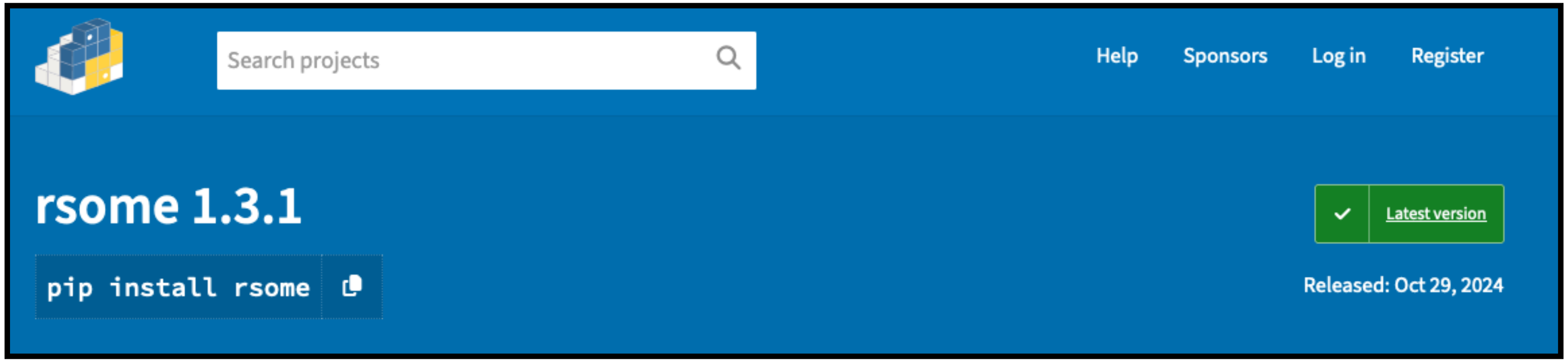
ACCESSIBILITY

Distributed under a free GPL V3.0 license with the support to a great variety of solvers.

Slide content credited to Peng Xiong

RSOME

- RSOME (Robust Stochastic Optimization Made Easy)
    - Installation



rsome 1.3.1

```
pip install rsome
```

Released: Oct 29, 2024

Robust Stochastic Optimization Made Easy

- Website: RSOME for Python
- PyPI: RSOME 1.3.1

Slide content credited to Peng Xiong

RSOME

- RSOME (Robust Stochastic Optimization Made Easy)
  - Solvers

Solver	License type	Required version	RSOME interface	Second-order cone constraints	Exponential cone constraints	Semidefiniteness constraints
scipy.optimize	Open-source	>= 1.9.0	`lpg_solver`	No	No	No
CyLP	Open-source	>= 0.9.0	`clp_solver`	No	No	No
OR-Tools	Open-source	>= 7.5.7466	`ort_solver`	No	No	No
ECOS	Open-source	>= 2.0.10	`eco_solver`	Yes	Yes	No
Gurobi	Commercial	>= 9.1.0	`grb_solver`	Yes	No	No
Mosek	Commercial	>= 10.0.44	`msk_solver`	Yes	Yes	Yes
CPLEX	Commercial	>= 12.9.0.0	`cpx_solver`	Yes	No	No
COPT	Commercial	>= 7.2.2	`cpt_solver`	Yes	Yes	Yes

COPT

- Solver

COPT (Cardinal Optimizer) is a mathematical optimization solver for large-scale problems. It is developed by Cardinal Operations.

2024 02月 **COPT 7.1** 混合整数规划和凸二次（约束）规划求解器性能进一步提升，新增支持GPU加速的一阶算法PDLP求解器

10月 **COPT 7.2** 混合整数规划等求解器性能再提升；新增支持指数锥，新增高性能自研C++矩阵建模功能，并且支持在Python接口启用

支持所有主流操作系统
Windows、MacOS、Linux（包括Apple Silicon和Arm64平台）

支持所有主流编程语言和第三方接口
Python、PuLP、Pyomo、CVXPY、Linopy、C、C++、C#、Julia、Java、AIMMS、AMPL、GAMS、RSOME、PyEPO、PyPSA和PyOptInterface等

获取杉数求解器

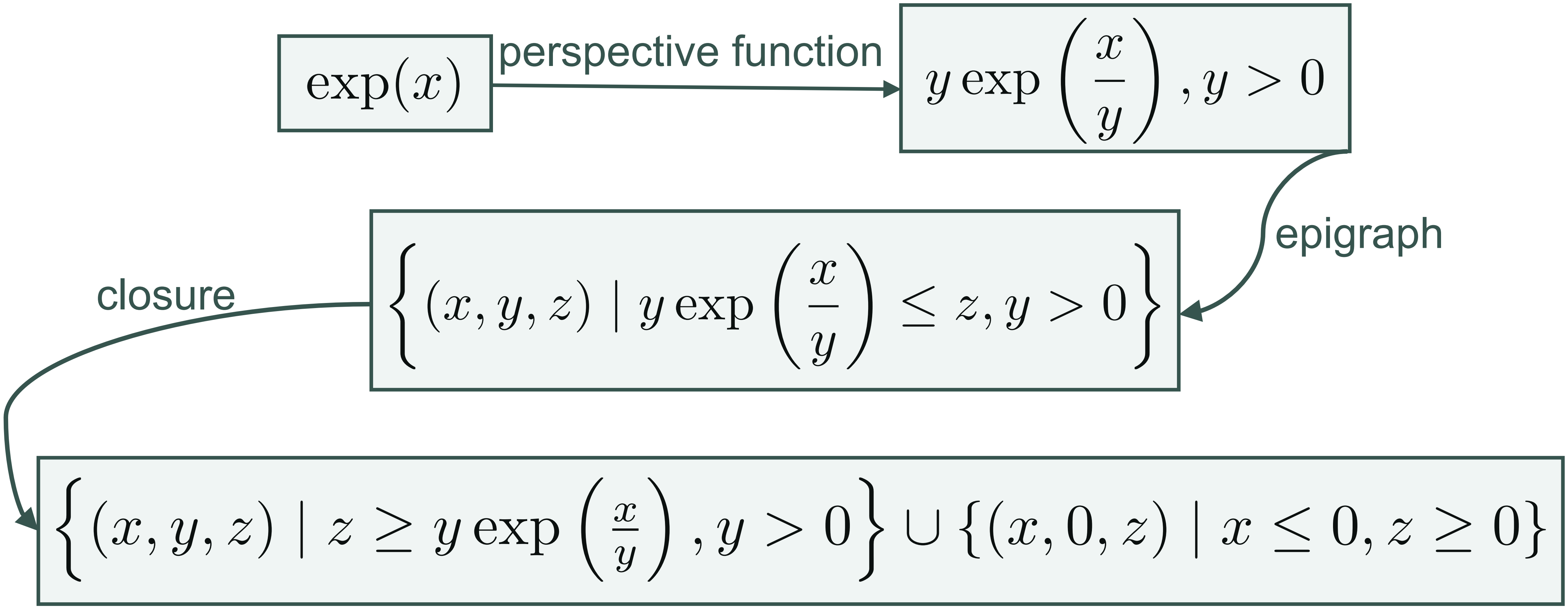
杉数求解器COPT 7.2 现已在全球范围内开放许可申请与下载。您可以填写申请表单，告诉我们相关信息。我们将审核用户申请表，**于2个工作日内将杉数求解器的安装包与免费授权许可可通过邮件发送给您**。针对学术用户，**请填写有效的学术邮箱**，我们提供365天的免费试用权限；针对非学术用户，我们提供至多180天的免费试用权限。杉数将持续推出更新版本的求解器试用套件，您可保持关注，继续申请获取。

本表格针对个人电脑的求解器申请试用。如需试用服务器版、浮动授权或计算集群版本，请与我们取得联系coptsales@shanshu.ai。

如果您在申请、使用求解器的过程中遇到任何问题，请写邮件到coptsupport@shanshu.ai或加入COPT求解器QQ群142636109讨论。

Exponential Cone

- Exponential cone: a 3-dim closed convex cone



$$\exp(x)$$

perspective function

$$y\exp\left(\frac{x}{y}\right), y > 0$$

epigraph

$$\left\{(x, y, z) \mid y\exp\left(\frac{x}{y}\right) \le z, y > 0\right\}$$

closure

$$\left\{(x, y, z) \mid z \ge y\exp\left(\frac{x}{y}\right), y > 0\right\} \cup \{(x, 0, z) \mid x \le 0, z \ge 0\}$$

MOSEK Modeling Cookbook
Release 3.3.0

MOSEK ApS

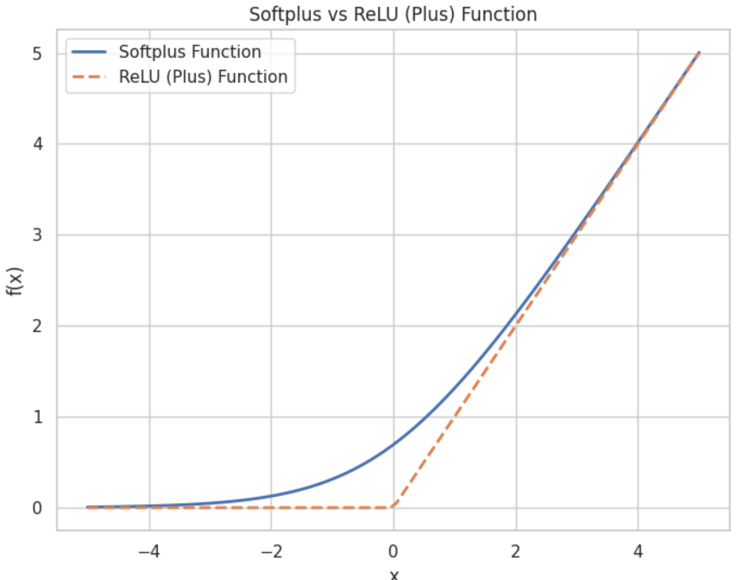
Exponential Cone

- Exponential cone

$$\mathcal{K}_{\exp} := \quad \{(x_1, x_2, x_3) \mid x_1 \geq x_2 \exp(x_3/x_2), x_2 > 0\} \cup \\ \{(x_1, 0, x_3) \mid x_1 \geq 0, x_3 \leq 0\}$$

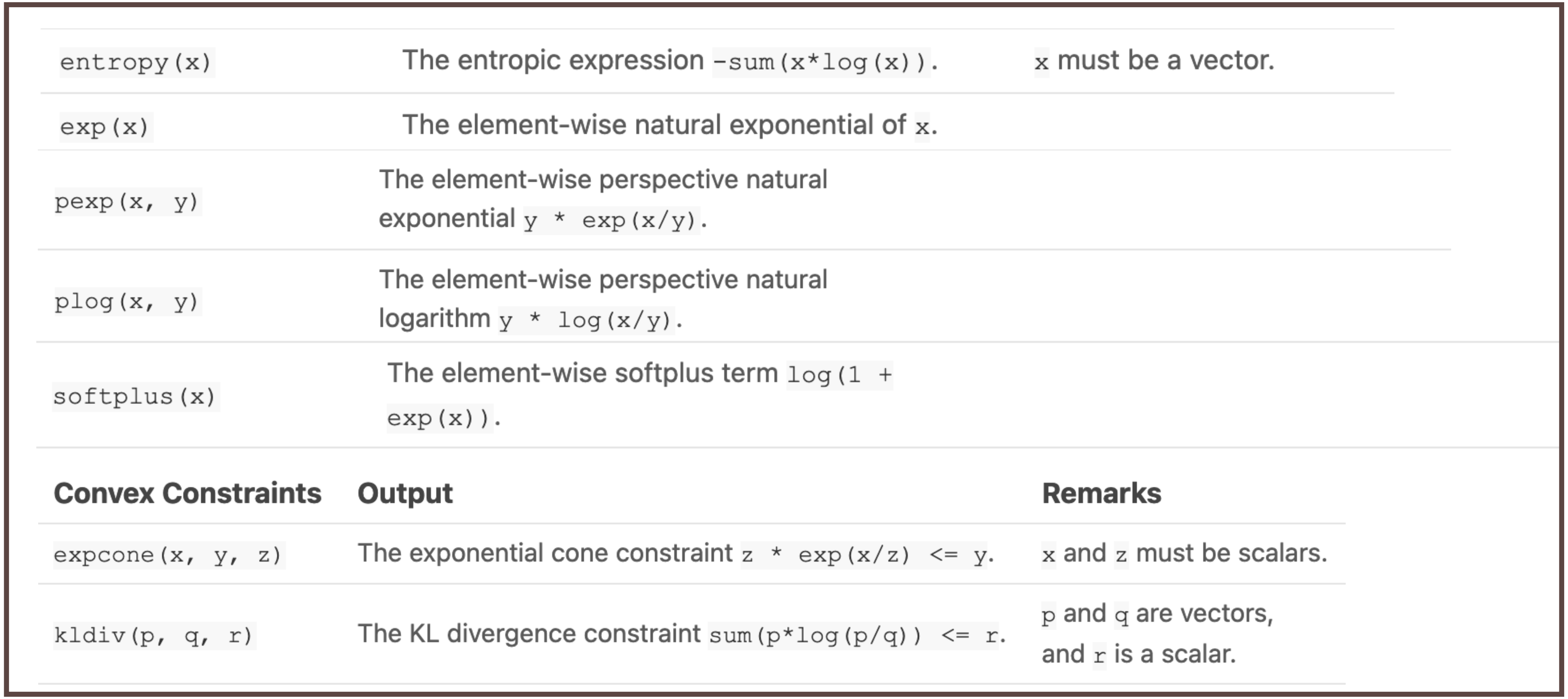
- Model exponential function: $t \geq \exp(x) \iff (t, 1, x) \in \mathcal{K}_{\exp}$
- Model softplus function:

$$\begin{aligned} & t \geq \log(1 + \exp(x)) \\ \iff \quad & \exp(t) \geq 1 + \exp(x) \\ \iff \quad & 1 \geq \exp(-t) + \exp(x - t) \\ \iff \quad & 1 \geq u + v \\ & (u, 1, -t) \in \mathcal{K}_{\exp} \\ & (v, 1, x - t) \in \mathcal{K}_{\exp} \end{aligned}$$



Exponential Cone

- Exponential cone related functions/constraints in RSOME



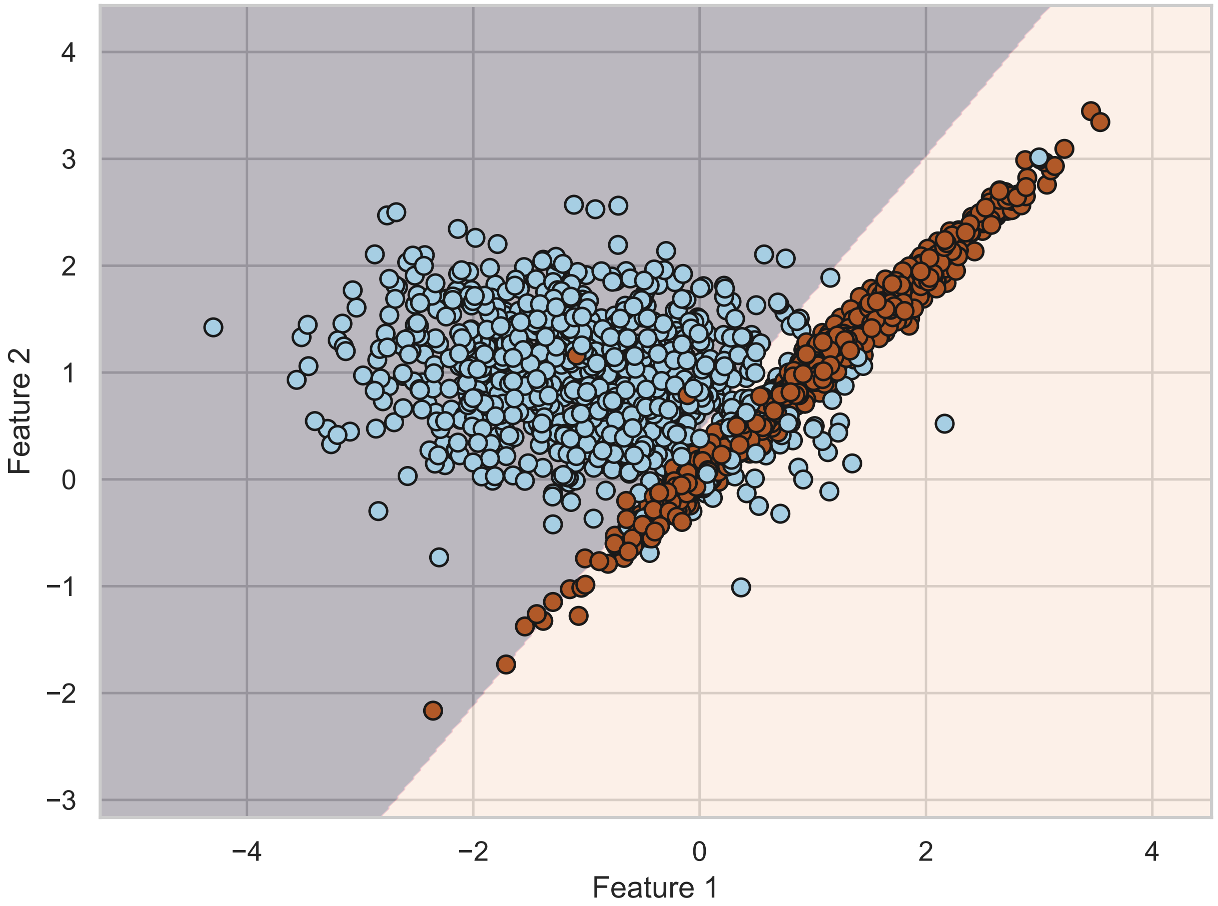
`entropy(x)`	The entropic expression `-sum(x*log(x))`.	`x` must be a vector.
`exp(x)`	The element-wise natural exponential of `x`.	
`pexp(x, y)`	The element-wise perspective natural exponential `y * exp(x/y)`.	
`plog(x, y)`	The element-wise perspective natural logarithm `y * log(x/y)`.	
`softplus(x)`	The element-wise softplus term `log(1 + exp(x))`.	

Convex Constraints	**Output**	**Remarks**
`expcone(x, y, z)`	The exponential cone constraint `z * exp(x/z) <= y`.	`x` and `z` must be scalars.
`kldiv(p, q, r)`	The KL divergence constraint `sum(p*log(p/q)) <= r`.	`p` and `q` are vectors, and `r` is a scalar.

Logistic Regression

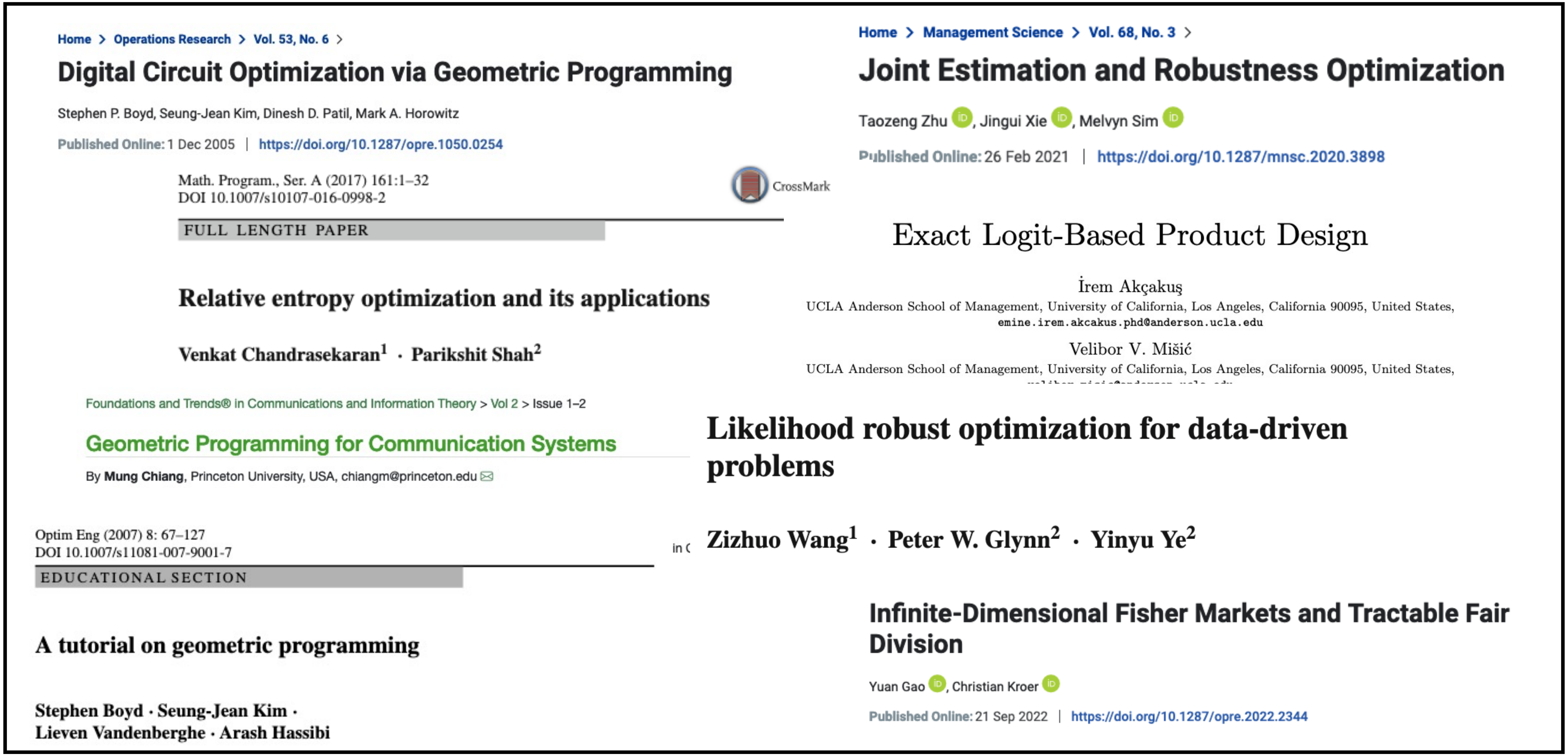
$$\min_{\boldsymbol{w},b} \ \frac{1}{n}\sum_{i=1}^{n}\log\left(1+\exp(-y_i(\boldsymbol{w}^\top \boldsymbol{x}_i+b))\right)$$

Logistic Regression Decision Boundary



```python
from rsome import ro
import rsome as rso
from rsome import cpt_solver as solver

lr = ro.Model()

w = lr.dvar(n_features)  # Weight vector
b = lr.dvar()            # Bias term
loss = lr.dvar(n_samples)

lr.st(loss >= rso.softplus(-y_train *
(X_train @ w + b) ) )

lr.min(sum(loss) * (1/n_samples))

lr.solve(solver)

w_opt = w.get()
b_opt = b.get()
```

Exponential Conic Optimization



Home > Operations Research > Vol. 53, No. 6 >

Digital Circuit Optimization via Geometric Programming

Stephen P. Boyd, Seung-Jean Kim, Dinesh D. Patil, Mark A. Horowitz

Published Online: 1 Dec 2005 | https://doi.org/10.1287/opre.1050.0254

Math. Program., Ser. A (2017) 161:1–32
DOI 10.1007/s10107-016-0998-2

CrossMark

FULL LENGTH PAPER

Relative entropy optimization and its applications

Venkat Chandrasekaran[1] · Parikshit Shah[2]

Foundations and Trends® in Communications and Information Theory > Vol 2 > Issue 1–2

Geometric Programming for Communication Systems

By **Mung Chiang**, Princeton University, USA, chiangm@princeton.edu ✉

Optim Eng (2007) 8: 67–127
DOI 10.1007/s11081-007-9001-7

EDUCATIONAL SECTION

A tutorial on geometric programming

Stephen Boyd · Seung-Jean Kim ·
Lieven Vandenberghe · Arash Hassibi

in (

Home > Management Science > Vol. 68, No. 3 >

Joint Estimation and Robustness Optimization

Taozeng Zhu, Jingui Xie, Melvyn Sim

Published Online: 26 Feb 2021 | https://doi.org/10.1287/mnsc.2020.3898

Exact Logit-Based Product Design

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Likelihood robust optimization for data-driven problems

Zizhuo Wang[1] · Peter W. Glynn[2] · Yinyu Ye[2]

Infinite-Dimensional Fisher Markets and Tractable Fair Division

Yuan Gao, Christian Kroer

Published Online: 21 Sep 2022 | https://doi.org/10.1287/opre.2022.2344

Optimization Under Uncertainty

- Deterministic Optimization

$$\min_{\boldsymbol{x} \in \mathcal{X}} f(\boldsymbol{x}, \boldsymbol{z})$$

- Stochastic Optimization

$$\min_{\boldsymbol{x} \in \mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]$$

- Robust Optimization

$$\min_{\boldsymbol{x} \in \mathcal{X}} \max_{\boldsymbol{z} \in \mathcal{Z}} f(\boldsymbol{x}, \boldsymbol{z})$$

- Distributionally Robust Optimization

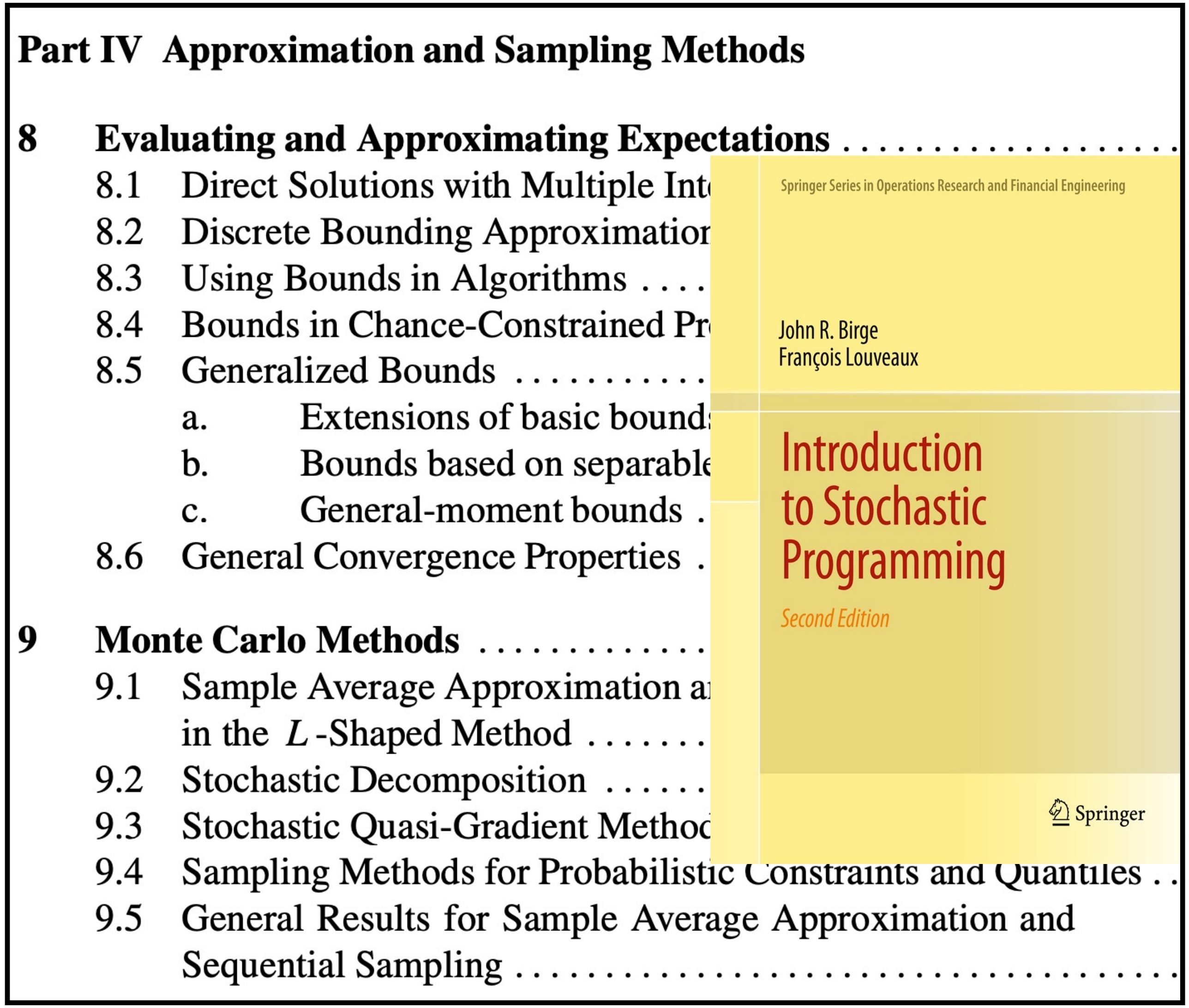
$$\min_{\boldsymbol{x} \in \mathcal{X}} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]$$

Stochastic Optimization

- Stochastic Optimization

$$\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x},\tilde{\boldsymbol{z}})\right]$$

  - Challenge: compute expectation
  - Two approaches
    - Deterministic approximation
    - Sample-based approximation



Part IV Approximation and Sampling Methods

Springer Series in Operations Research and Financial Engineering

John R. Birge
François Louveaux

Introduction to Stochastic Programming

Second Edition

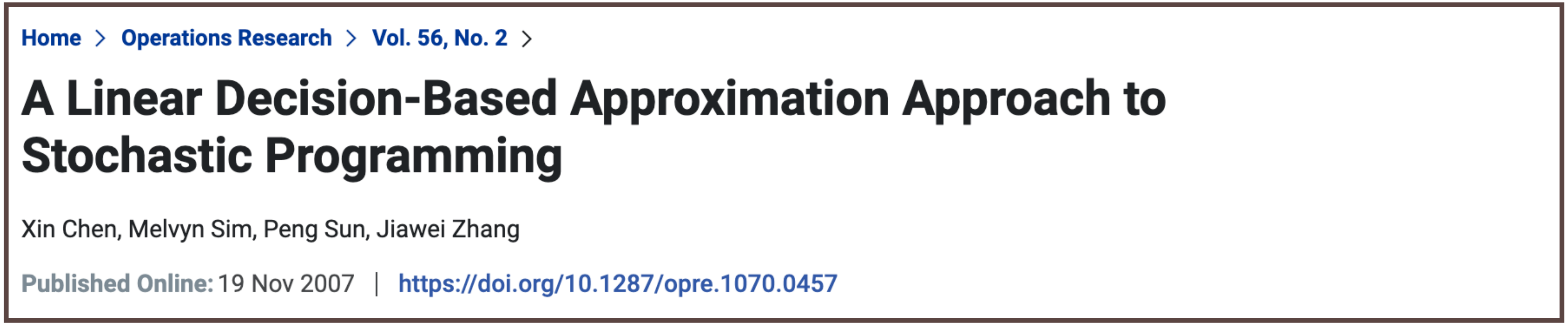
Springer

Optimization Under Uncertainty

- Stochastic Optimization
  - Challenge: compute expectation
- Robust perspective: approximate expectation by upper bounds

$$\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x},\tilde{\boldsymbol{z}})\right]$$

$$\mathbb{P}\in\mathcal{F} \implies \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x},\tilde{\boldsymbol{z}})\right] \leq \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x},\tilde{\boldsymbol{z}})\right]$$



Home > Operations Research > Vol. 56, No. 2 >

A Linear Decision-Based Approximation Approach to Stochastic Programming

Xin Chen, Melvyn Sim, Peng Sun, Jiawei Zhang

Published Online: 19 Nov 2007 | https://doi.org/10.1287/opre.1070.0457

Optimization Under Uncertainty

- Robust perspective: approximate expectation by upper bounds

$$\boxed{\mathbb{P} \in \mathcal{F} \Longrightarrow \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right] \leq \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]}$$

- More information ⇒ smaller ambiguity set ⇒ less conservative
- Modeling idea: leverage the moment-generating function (MGF)

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Distributionally Robust Optimization with Infinitely Constrained Ambiguity Sets

Zhi Chen, Melvyn Sim, Huan Xu

Published Online: 5 Jul 2019 | https://doi.org/10.1287/opre.2018.1799

Optimization Under Uncertainty

- Robust perspective: approximate expectation by upper bounds

$$\boxed{\mathbb{P} \in \mathcal{F} \implies \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right] \leq \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]}$$

- Modeling idea: leverage the moment-generating function (MGF)
- Tractability: exponential conic optimization

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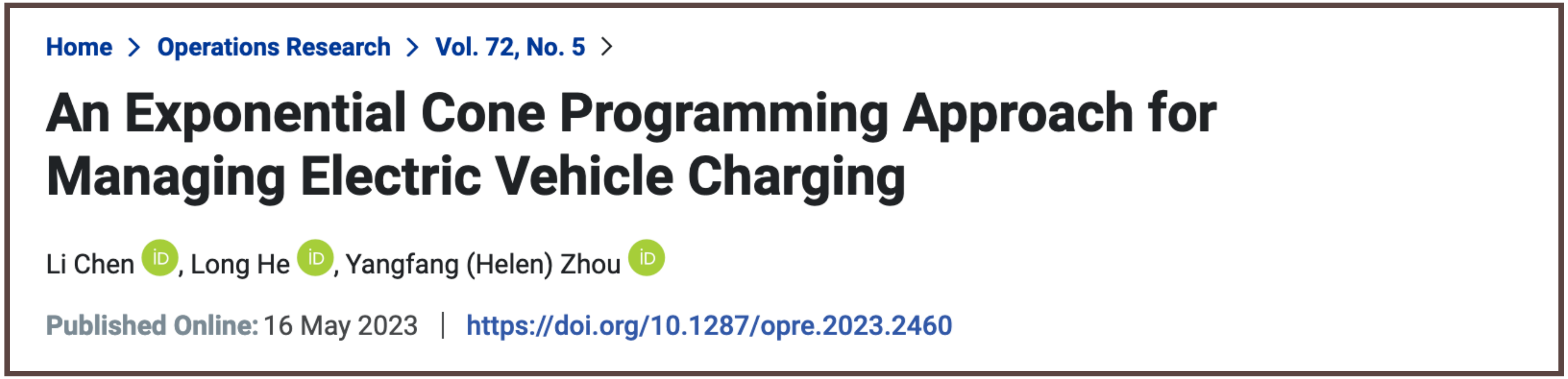
An Exponential Cone Programming Approach for Managing Electric Vehicle Charging

Li Chen, Long He, Yangfang (Helen) Zhou

Published Online: 16 May 2023 | https://doi.org/10.1287/opre.2023.2460

Part II



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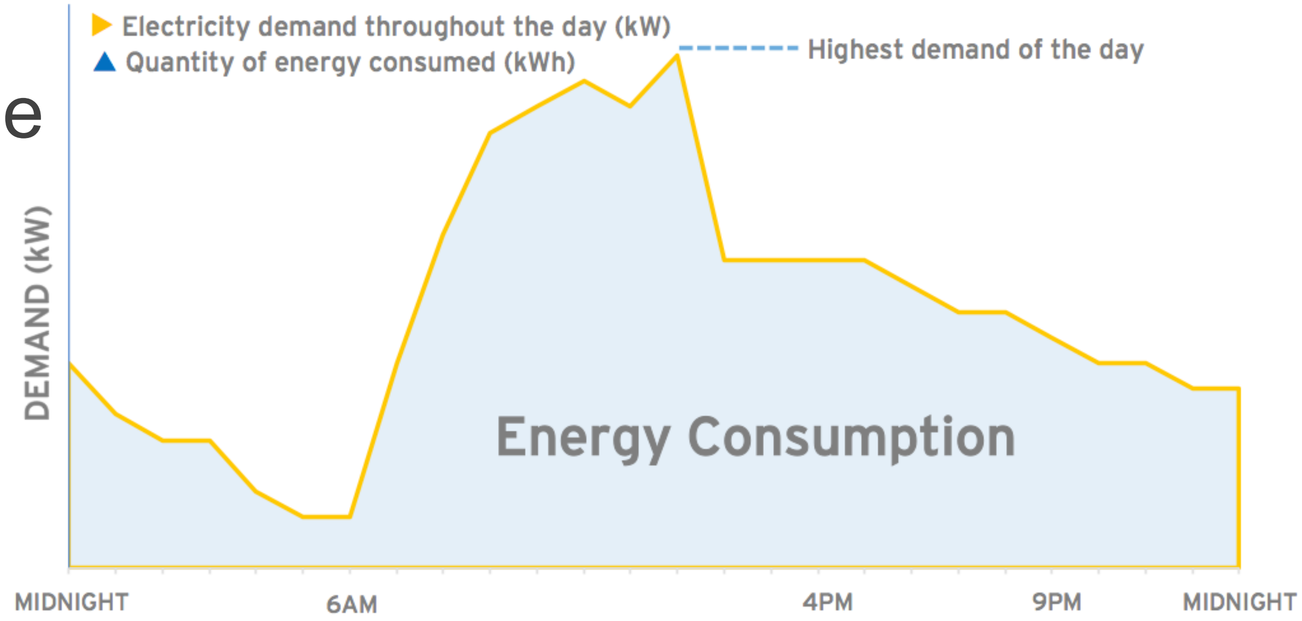
An Exponential Cone Programming Approach for Managing Electric Vehicle Charging

Li Chen, Long He, Yangfang (Helen) Zhou

Published Online: 16 May 2023 | https://doi.org/10.1287/opre.2023.2460

Problem Background

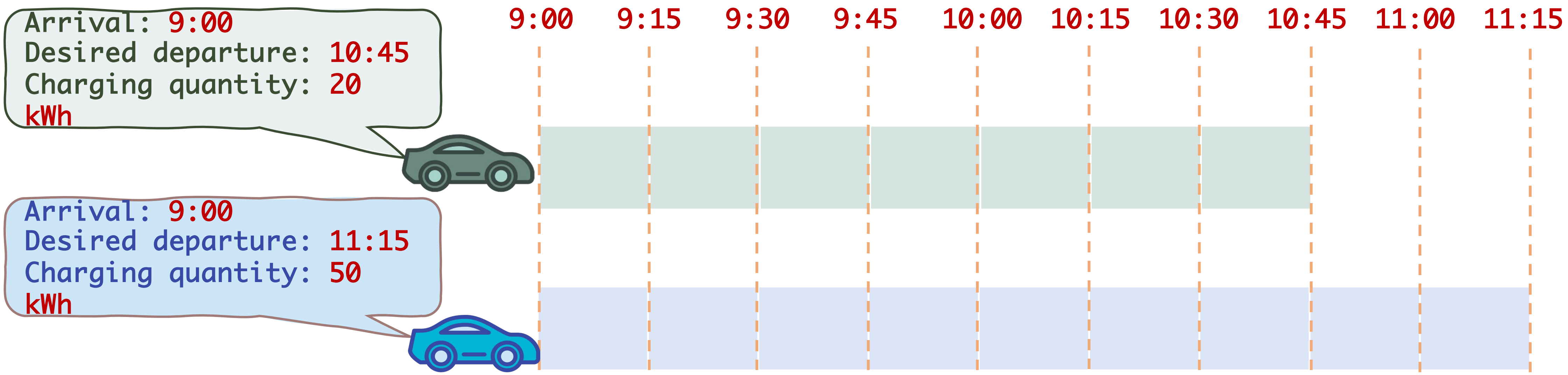
- Stochastic demand of heterogeneous customers
  - arrival time
  - departure time
  - energy requirement
- Decision: charging schedule
- Electricity tariff structure
  - energy charge
  - demand charge
- Goal: minimize cost



Source: https://www.sdge.com/businesses/pricing-plans/understanding-demand

Problem Description

9:00 am

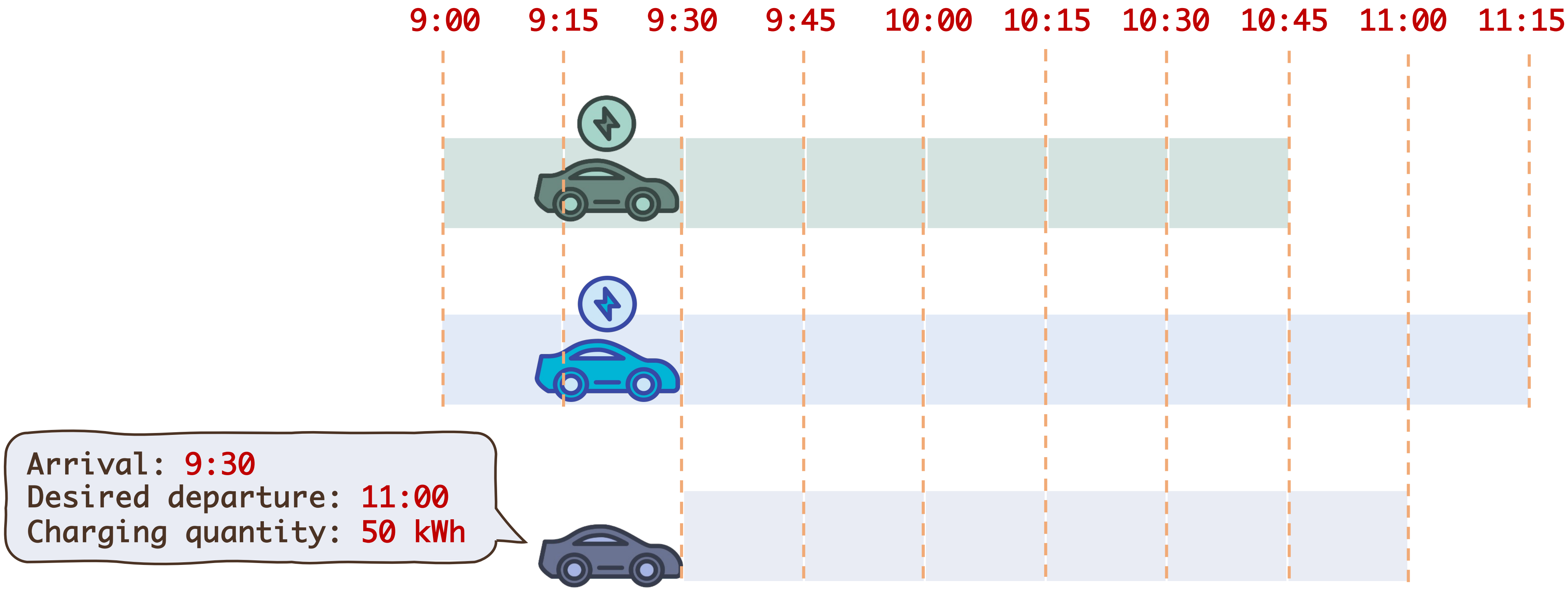


Arrival: 9:00
Desired departure: 10:45
Charging quantity: 20 kWh

Arrival: 9:00
Desired departure: 11:15
Charging quantity: 50 kWh

Problem Description

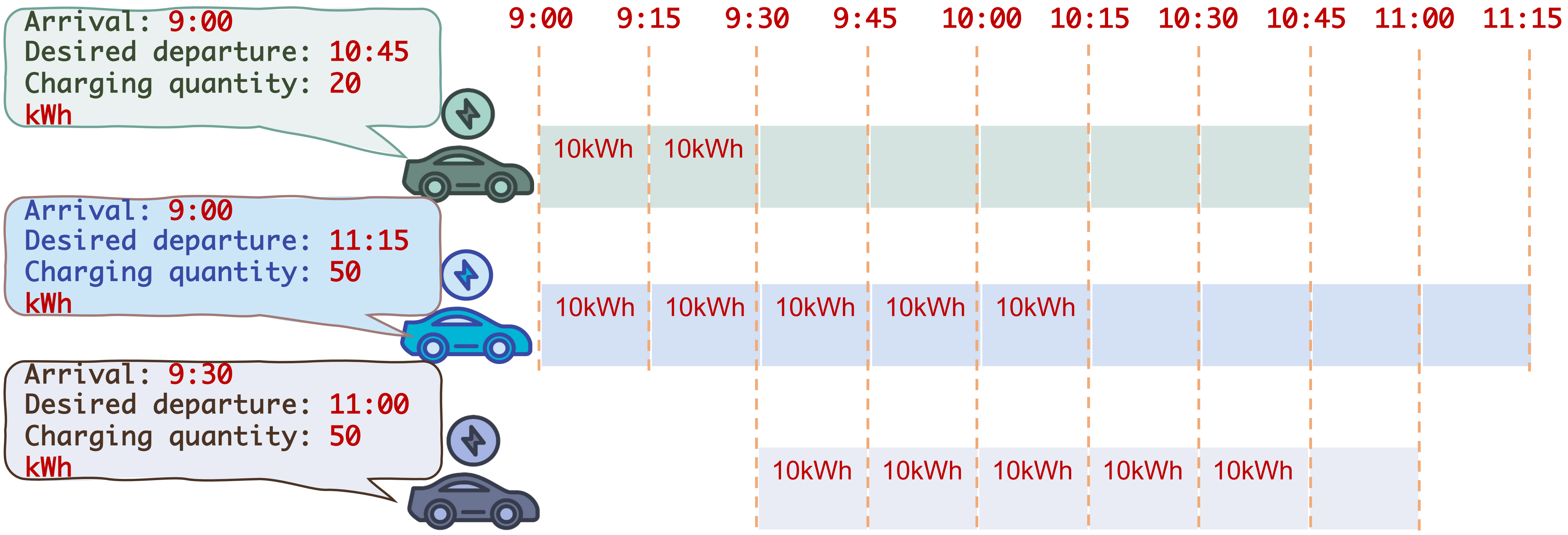
9:30 am



Arrival: 9:30
Desired departure: 11:00
Charging quantity: 50 kWh

Problem Description

Energy cost/ kWh	$0.1	$0.1	$0.1	$0.1	$0.2	$0.2	$0.2	$0.2	$0.2



Arrival: 9:00
Desired departure: 10:45
Charging quantity: 20 kWh

Arrival: 9:00
Desired departure: 11:15
Charging quantity: 50 kWh

Arrival: 9:30
Desired departure: 11:00
Charging quantity: 50 kWh

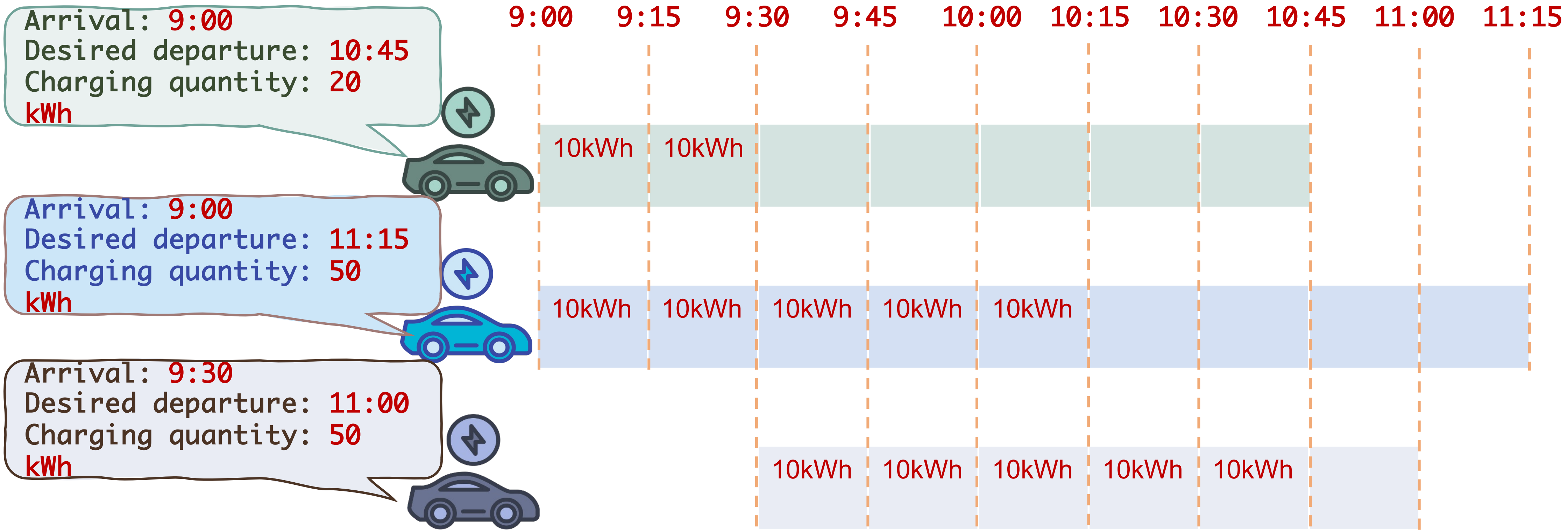
Energy consumption	20kWh	20kWh	20kWh	20kWh	20kWh	10kWh	10kWh	0kWh	0kWh

Problem Description

Demand charge: $1/ kW

Cost = $16 (energy charge) + $80 (demand charge)

Energy cost/ kWh	$0.1	$0.1	$0.1	$0.1	$0.2	$0.2	$0.2	$0.2	$0.2



Arrival: 9:00
Desired departure: 10:45
Charging quantity: 20 kWh

Arrival: 9:00
Desired departure: 11:15
Charging quantity: 50 kWh

Arrival: 9:30
Desired departure: 11:00
Charging quantity: 50 kWh

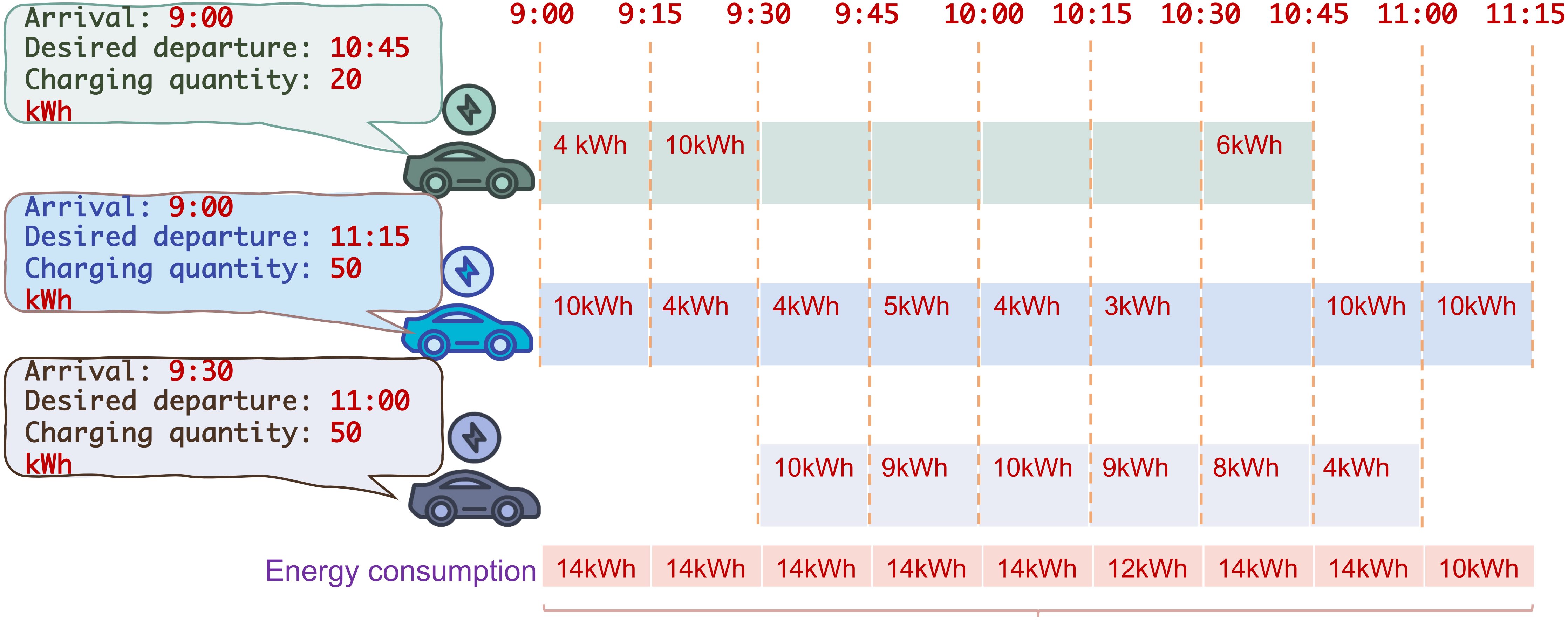
Energy consumption	20kWh	20kWh	20kWh	20kWh	20kWh	10kWh	10kWh	0kWh	0kWh

Peak load: 80 kW

Problem Description

Demand charge: $1/ kW

Cost = $18.4 (energy charge) + $56 (demand charge)



Arrival: 9:00
Desired departure: 10:45
Charging quantity: 20 kWh

Arrival: 9:00
Desired departure: 11:15
Charging quantity: 50 kWh

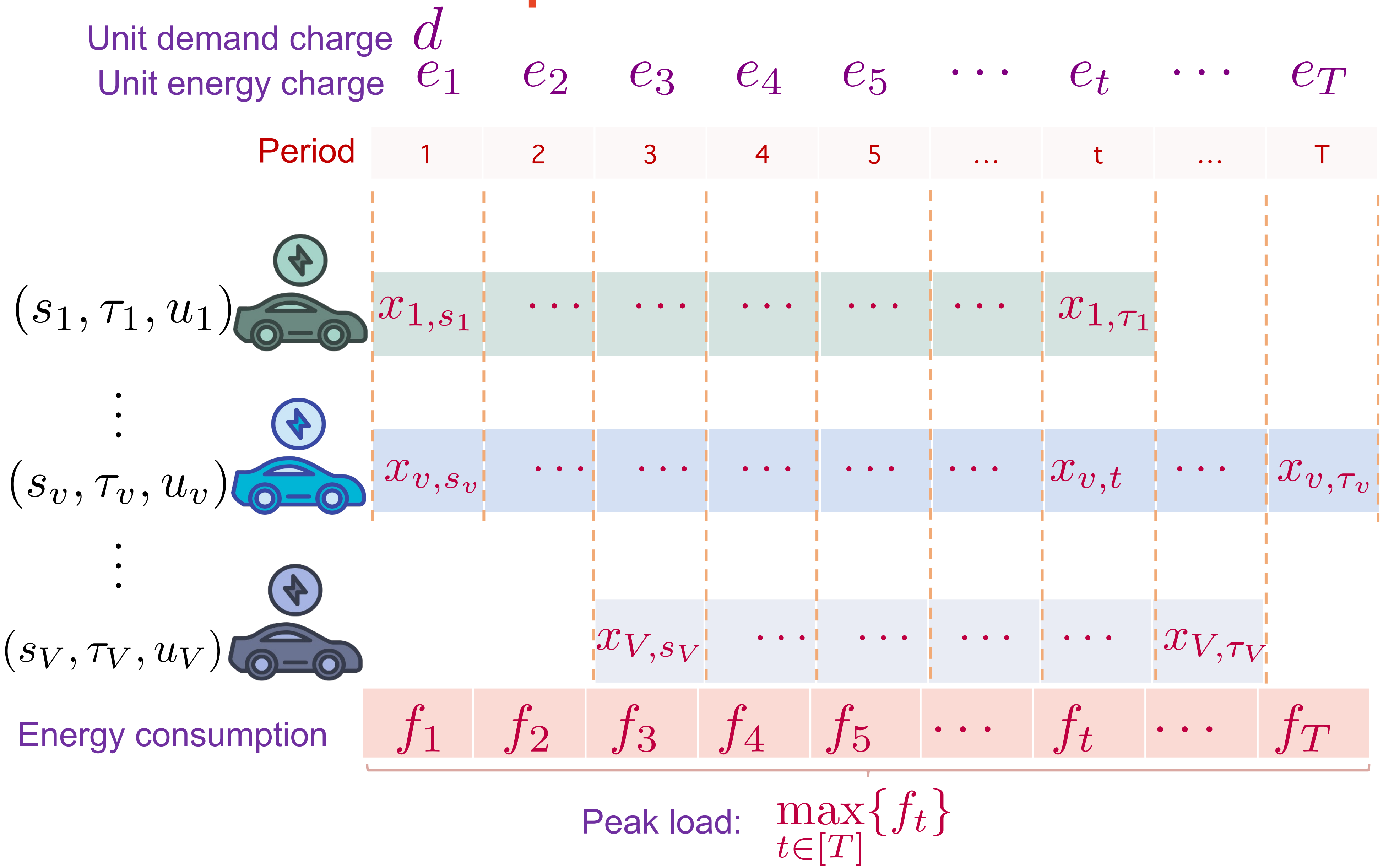
Arrival: 9:30
Desired departure: 11:00
Charging quantity: 50 kWh

9:00 9:15 9:30 9:45 10:00 10:15 10:30 10:45 11:00 11:15

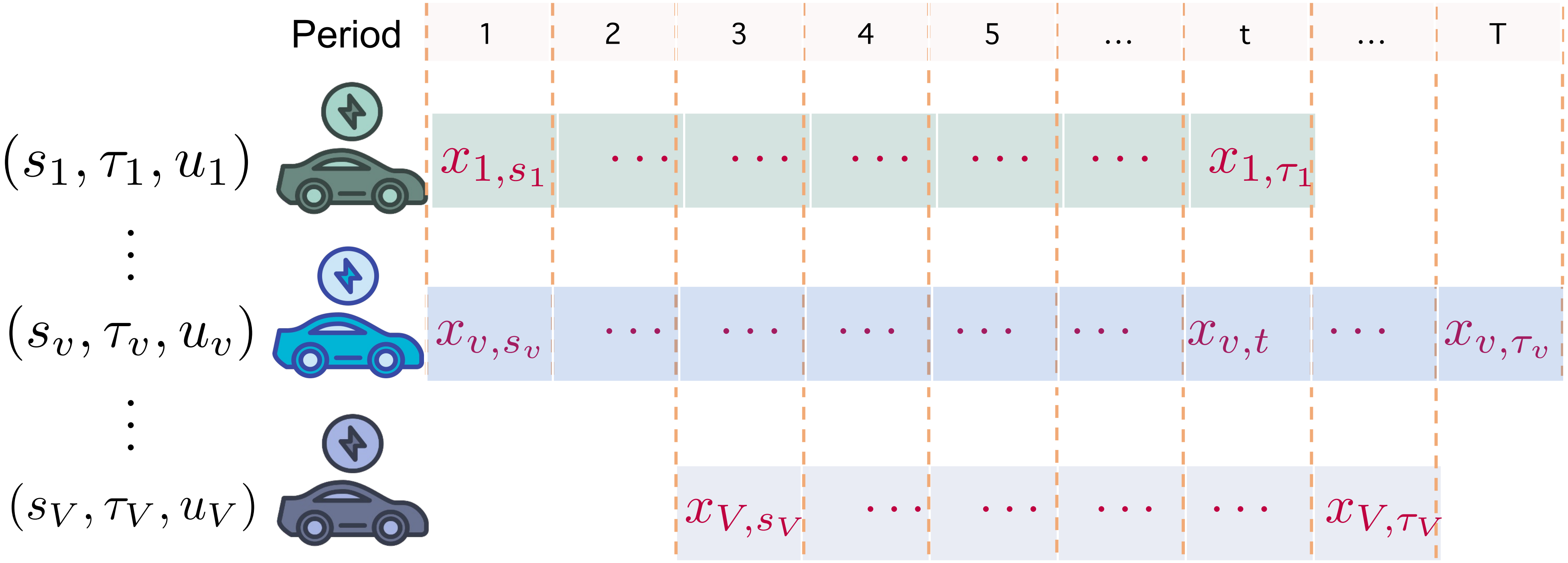
Energy cost/ kWh	$0.1	$0.1	$0.1	$0.1	$0.2	$0.2	$0.2	$0.2	$0.2
	4 kWh	10kWh					6kWh		
	10kWh	4kWh	4kWh	5kWh	4kWh	3kWh		10kWh	10kWh
			10kWh	9kWh	10kWh	9kWh	8kWh	4kWh	
Energy consumption	14kWh	14kWh	14kWh	14kWh	14kWh	12kWh	14kWh	14kWh	10kWh

Peak load: 56 kW

Problem Description



Problem Description



Charging Constraints

$$\sum_{t\in\mathcal{T}_v} \eta x_{v,t} = u_v \quad \forall v \in [V]$$

$$0 \leq x_{v,t} \leq K/\eta \quad \forall v \in [V], t \in \mathcal{T}_v$$

The Stochastic Program

$$\boxed{\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) + d\max_{t\in[T]}\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\}\right]}$$

The Stochastic Program

$$\boxed{\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) + d\max_{t\in[T]}\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\}\right]}$$

$$\mathcal{X} := \left\{ \boldsymbol{x} \;\middle|\; \begin{array}{ll} \sum_{t\in\mathcal{T}_v} \eta x_{v,t} = u_v & \forall v\in[V] \\ 0\le x_{v,t}\le K/\eta & \forall v\in[V], t\in\mathcal{T}_v \end{array} \right\}.$$

Feasible charging schedule

The Stochastic Program

$$\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) + d\max_{t\in[T]}\left\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\right\}\right]$$

$$f_t(\boldsymbol{x},\boldsymbol{z}) = \sum_{v\in\mathcal{V}_t} x_{v,t} z_v$$

Energy consumption in period t

The Stochastic Program

$$\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) + d\max_{t\in[T]}\left\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\right\}\right]$$

$$f_t(\boldsymbol{x},\boldsymbol{z}) = \sum_{v\in\mathcal{V}_t} x_{v,t} z_v$$

Number of arrivals of type v

All the types for charging in period t

$$\mathcal{V}_t := \{v \in [V] : s_v \le t \le \tau_v\}$$

The Stochastic Program

$$\min_{\boldsymbol{x}\in\mathcal{X}} \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) + d\max_{t\in[T]}\left\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\right\}\right]$$

- Given arrival distributions $\tilde{\boldsymbol{z}} \sim \mathbb{P}$ such as independent Poisson arrivals, one can use sample average approximation (SAA)

$$\min_{\boldsymbol{x}\in\mathcal{X}} \frac{1}{S}\sum_{s\in[S]}\left(\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\hat{\boldsymbol{z}}_s) + d\max_{t\in[T]}\left\{f_t(\boldsymbol{x},\hat{\boldsymbol{z}}_s)\right\}\right)$$

The Stochastic Program

- Sample average approximation (SAA)

$$\min_{\boldsymbol{x}\in\mathcal{X}} \frac{1}{S} \sum_{s\in[S]} \left(\sum_{t\in[T]} e_t f_t(\boldsymbol{x}, \hat{\boldsymbol{z}}_s) + d \max_{t\in[T]} \{ f_t(\boldsymbol{x}, \hat{\boldsymbol{z}}_s) \} \right)$$

- Large scale: (>80000) random variables and (>700000) decision variables, and large sample size (>5000).
- Alternative: a robust perspective using MGF

Exponential Conic Approximation

$$\mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} e_t f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) + d\max_{t\in[T]}\left\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\right\}\right]$$

$$= \sum_{t\in[T]} e_t f_t(\boldsymbol{x},\boldsymbol{\lambda}) + d\mathbb{E}_{\mathbb{P}}\left[\max_{t\in[T]}\left\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\right\}\right]$$

$$\mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] = \boldsymbol{\lambda} \Longrightarrow \mathbb{E}_{\mathbb{P}}\left[f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\right] = f_t(\boldsymbol{x},\boldsymbol{\lambda})$$

Exponential Conic Approximation

- Bound the expected peak load using MGF:

$$
\begin{aligned}
&\mathbb{E}_{\mathbb{P}}\left[\max_{t\in[T]}\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\}\right] \\
&\leq \inf_{\mu>0} \mu \log \sum_{t\in[T]} \mathbb{E}_{\mathbb{P}}\left[\exp\left(\frac{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}}) - f_t(\boldsymbol{x},\boldsymbol{\lambda})}{\mu}\right)\right] + \max_{t\in[T]}\{f_t(\boldsymbol{x},\boldsymbol{\lambda})\}
\end{aligned}
$$

Exponential Conic Approximation

- Bound the expected peak load using MGF:

$$\begin{aligned}&\mathbb{E}_{\mathbb{P}}\left[\max_{t\in[T]}\{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})\}\right]\\ \leq\ &\inf_{\mu>0}\mu\log\sum_{t\in[T]}\mathbb{E}_{\mathbb{P}}\left[\exp\left(\frac{f_t(\boldsymbol{x},\tilde{\boldsymbol{z}})-f_t(\boldsymbol{x},\boldsymbol{\lambda})}{\mu}\right)\right]+\max_{t\in[T]}\{f_t(\boldsymbol{x},\boldsymbol{\lambda})\}\end{aligned}$$

- Exploit stochastic independence to decompose log-MGF:

$$\begin{aligned}&\mu\log\mathbb{E}_{\mathbb{P}}\left[\exp\left(\frac{\sum_{v\in\mathcal{V}_t}x_{v,t}\tilde{z}_v-\sum_{v\in\mathcal{V}_t}x_{v,t}\lambda_v}{\mu}\right)\right]\\ =&\sum_{v\in\mathcal{V}_t}\mu\log\mathbb{E}_{\mathbb{P}}\left[\exp\left(\frac{x_{v,t}\tilde{z}_v}{\mu}\right)\right]-\sum_{v\in\mathcal{V}_t}x_{v,t}\lambda_v\end{aligned}$$

Exponential Conic Approximation

- Bound the expected peak load using MGF:
- Exploit stochastic independence to decompose log-MGF:

$$\mu \log \mathbb{E}_{\mathbb{P}} \left[\exp \left(\frac{\sum_{v \in \mathcal{V}_t} x_{v,t} \tilde{z}_v - \sum_{v \in \mathcal{V}_t} x_{v,t} \lambda_v}{\mu} \right) \right]$$
$$= \sum_{v \in \mathcal{V}_t} \mu \log \mathbb{E}_{\mathbb{P}} \left[\exp \left(\frac{x_{v,t} \tilde{z}_v}{\mu} \right) \right] - \sum_{v \in \mathcal{V}_t} x_{v,t} \lambda_v$$

- Exponential cone representable log-MGF of Poisson variables:

$$\mu \log \mathbb{E}_{\mathbb{P}} \left[\exp \left(\frac{x_{v,t} \tilde{z}_v}{\mu} \right) \right] = \lambda_v (\mu e^{x_{v,t}/\mu} - \mu) \text{ is } \mathcal{K}_{\exp}\text{-representable}$$

Exponential Conic Approximation

- Exponential conic representable upper bound:

$$
\text{(ECP-U)} \qquad
\begin{aligned}
\inf_{\substack{\boldsymbol{x}\in\mathcal{X},\kappa,\gamma,\\ \mu>0,\boldsymbol{\xi},\boldsymbol{\zeta}}} \quad & d\left(\kappa+\gamma\right)+\sum_{t\in[T]} e_t\left(\sum_{v\in\mathcal{V}_t} x_{v,t}\lambda_v\right) & \\
\text{s.t.} \quad & \sum_{v\in\mathcal{V}_t} x_{v,t}\lambda_v \leq \gamma & \forall t\in[T] \\
& \mu\exp\left(x_{v,t}/\mu\right)\leq \xi_{v,t} & \forall t\in[T], v\in\mathcal{V}_t \\
& \mu\exp\left(\left(-\kappa+\sum_{v\in\mathcal{V}_t}\lambda_v(\xi_{v,t}-x_{v,t}-\mu)\right)/\mu\right)\leq \zeta_t & \forall t\in[T] \\
& \sum_{t\in[T]}\zeta_t\leq\mu &
\end{aligned}
$$

Exponential Conic Approximation

- Exponential conic constraints:

$$\mu \exp\left(x_{v,t}/\mu\right) \leq \xi_{v,t} \qquad \forall t \in [T], v \in \mathcal{V}_t$$

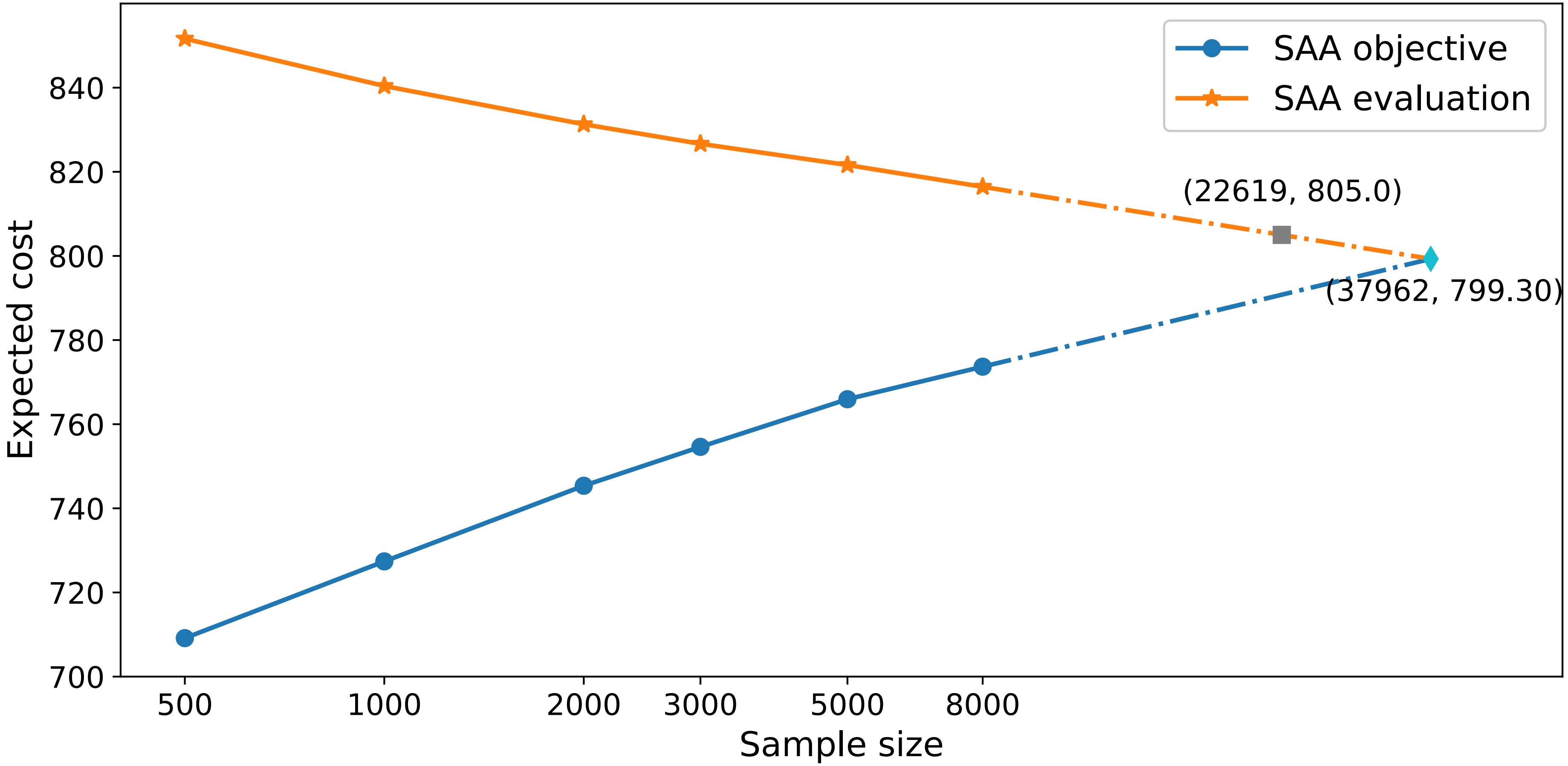
```
xi[v, t] >= rso.pexp(x[v, t], mu)
```

$$\mu \exp\left(\left(-\kappa + \sum_{v \in \mathcal{V}_t} \lambda_v(\xi_{v,t} - x_{v,t} - \mu)\right)/\mu\right) \leq \zeta_t \qquad \forall t \in [T]$$

```
zeta[t] >= rso.pexp(-kappa + sum((xi[v, t] - x[v, t] - mu) * v_rates[v]
                                 for v in vt_types[t]), mu)
```

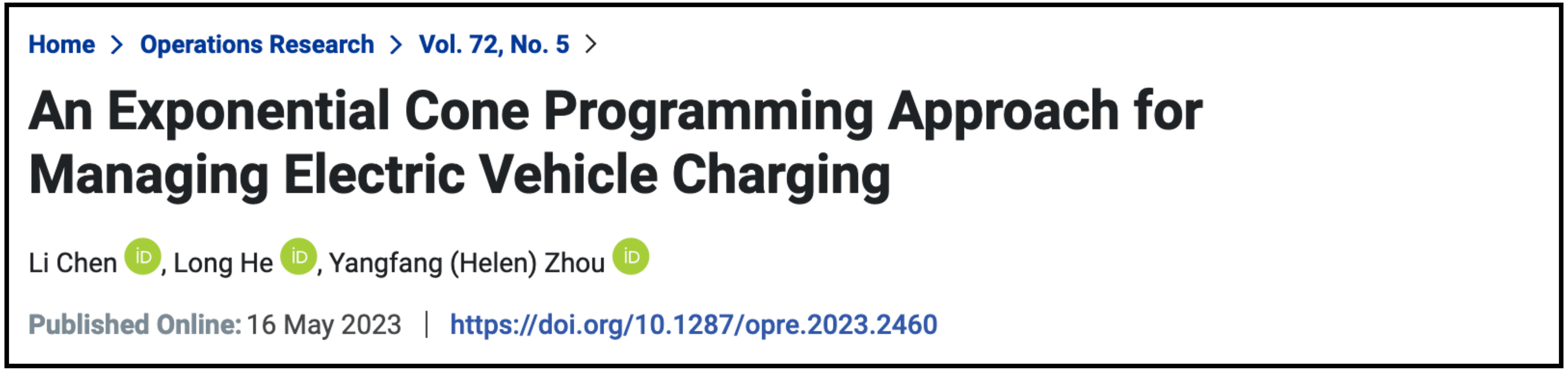
Comparison with SAA

- SAA with 38000 samples: >36 hours, far from optimality
- ECP is near optimal: optimality gap ≈ 0.71%



Key Ideas

- Modeling
  - Incorporate more distributional information via MGF
  - Exploit stochastic independence via MGF
- Computation
  - Computational scalability by ECP approximation



Home > Operations Research > Vol. 72, No. 5 >

An Exponential Cone Programming Approach for Managing Electric Vehicle Charging

Li Chen, Long He, Yangfang (Helen) Zhou

Published Online: 16 May 2023 | https://doi.org/10.1287/opre.2023.2460

Part III

- A general framework

Home > Operations Research > Ahead of Print >

Robust Optimization with Moment-Dispersion Ambiguity

Li Chen, Chenyi Fu, Fan Si, Melvyn Sim, Peng Xiong

Published Online: 16 Dec 2024 | https://doi.org/10.1287/opre.2023.0579

Make It General

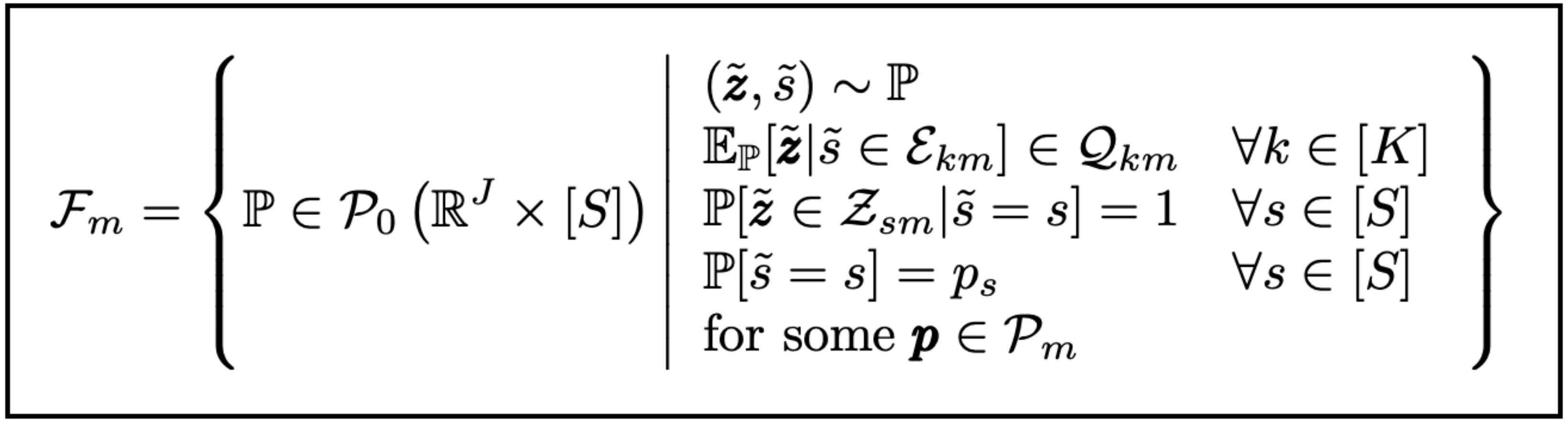
- How to generalize these ideas and integrate with existing RSOME framework?

Home > Management Science > Vol. 66, No. 8 >

Robust Stochastic Optimization Made Easy with RSOME

Zhi Chen, Melvyn Sim, Peng Xiong

Published Online: 13 May 2020 | https://doi.org/10.1287/mnsc.2020.3603



$$\mathcal{F}_m = \left\{ \mathbb{P} \in \mathcal{P}_0\left(\mathbb{R}^J \times [S]\right) \;\middle|\; \begin{array}{ll} (\tilde{\boldsymbol{z}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{z}} | \tilde{s} \in \mathcal{E}_{km}] \in \mathcal{Q}_{km} & \forall k \in [K] \\ \mathbb{P}[\tilde{\boldsymbol{z}} \in \mathcal{Z}_{sm} | \tilde{s} = s] = 1 & \forall s \in [S] \\ \mathbb{P}[\tilde{s} = s] = p_s & \forall s \in [S] \\ \text{for some } \boldsymbol{p} \in \mathcal{P}_m & \end{array} \right\}$$

Make It General

- The building block: linear moment ambiguity sets

$$\mathcal{F}_m = \left\{ \mathbb{P} \in \mathcal{P}_0\left(\mathbb{R}^J \times [S]\right) \;\middle|\; \begin{array}{ll} (\tilde{\boldsymbol{z}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{z}} \cancel{| \tilde{s} \in \mathcal{E}_{km}]} \in \mathcal{Q}_{km} & \forall k \in [K] \\ \mathbb{P}[\tilde{\boldsymbol{z}} \in \mathcal{Z}_{sm} \cancel{| \tilde{s} = s]} = 1 & \forall s \in [S] \\ \cancel{\mathbb{P}[\tilde{s} = s] = p_s} & \cancel{\forall s \in [S]} \\ \cancel{\text{for some } \boldsymbol{p} \in \mathcal{P}_m} & \end{array} \right\}$$

$$S = 1$$

$$\mathcal{F}_L := \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{N_z}) \;\middle|\; \begin{array}{l} \tilde{\boldsymbol{z}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] \in \mathcal{W} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \in \mathcal{Z}\right] = 1 \end{array} \right\}$$

Moment-Dispersion Ambiguity

Mean uncertainty set

$$\mathcal{G} := \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{N_z}) \,\middle|\, \begin{array}{l} \tilde{\boldsymbol{z}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] \in \boxed{\mathcal{W}} \\ \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\boldsymbol{v}^\top(\tilde{\boldsymbol{z}} - \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right])\right)\right] \leq \boxed{\varphi(\boldsymbol{v})} \quad \forall \boldsymbol{v} \in \mathbb{R}^{N_z} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \in \boxed{\mathcal{Z}}\right] = 1 \end{array} \right\}$$

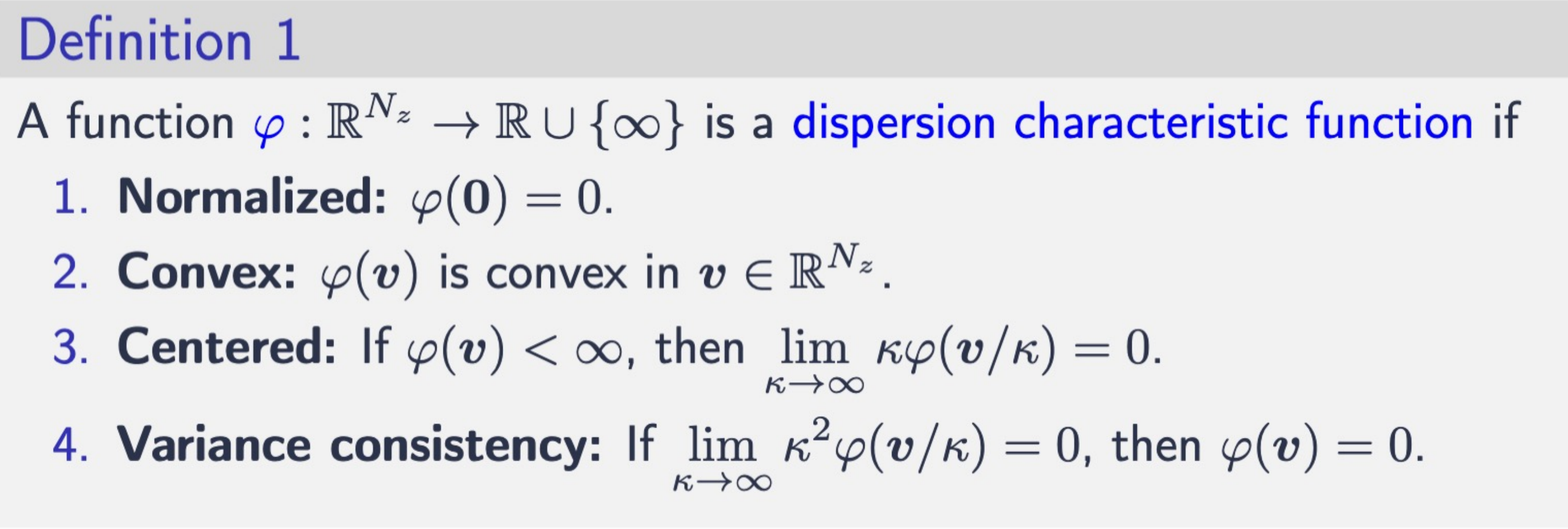
Support set

Dispersion characteristic function

Moment-Dispersion Ambiguity

- Moment-dispersion ambiguity sets

$$\mathcal{G} := \left\{ \mathbb{P} \in \mathcal{P}_0\left(\mathbb{R}^{N_z}\right) \;\middle|\; \begin{array}{ll} \tilde{\boldsymbol{z}} \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] \in \mathcal{W} & \\ \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\boldsymbol{v}^\top\left(\tilde{\boldsymbol{z}} - \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right]\right)\right)\right] \leq \boxed{\varphi(\boldsymbol{v})} & \forall \boldsymbol{v} \in \mathbb{R}^{N_z} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \in \mathcal{Z}\right] = 1 & \end{array} \right\}$$



Definition 1

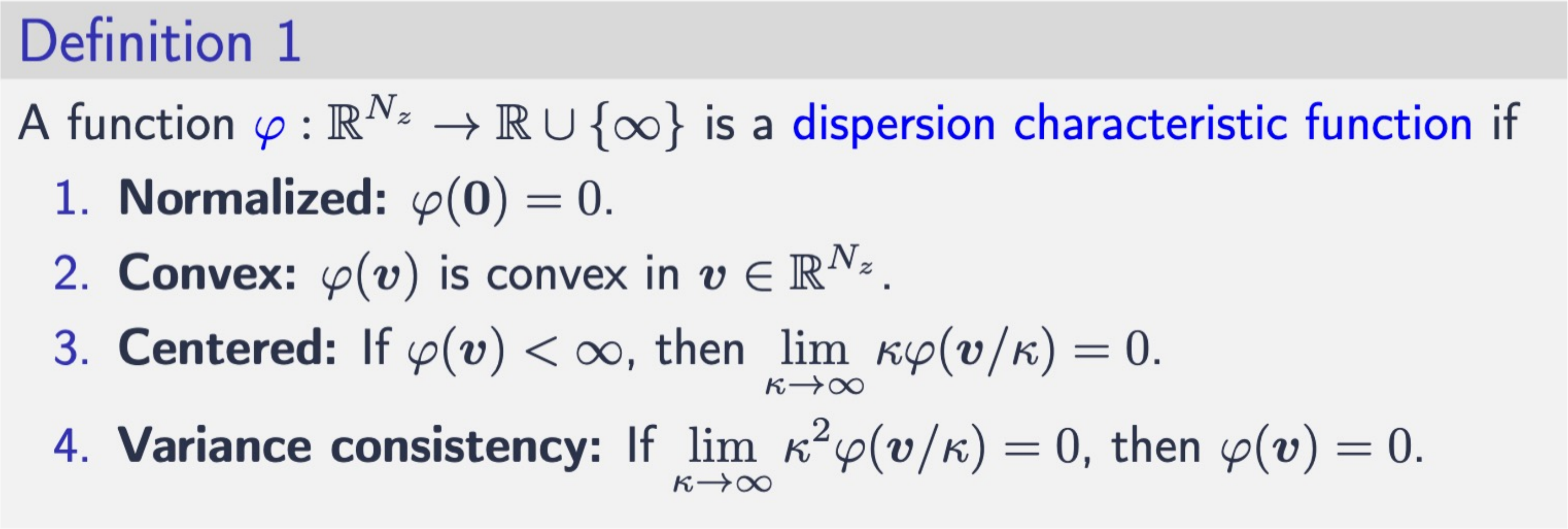
A function $\varphi : \mathbb{R}^{N_z} \rightarrow \mathbb{R} \cup \{\infty\}$ is a dispersion characteristic function if

1. **Normalized:** $\varphi(\mathbf{0}) = 0$.
2. **Convex:** $\varphi(\boldsymbol{v})$ is convex in $\boldsymbol{v} \in \mathbb{R}^{N_z}$.
3. **Centered:** If $\varphi(\boldsymbol{v}) < \infty$, then $\lim_{\kappa \rightarrow \infty} \kappa\varphi(\boldsymbol{v}/\kappa) = 0$.
4. **Variance consistency:** If $\lim_{\kappa \rightarrow \infty} \kappa^2\varphi(\boldsymbol{v}/\kappa) = 0$, then $\varphi(\boldsymbol{v}) = 0$.

Moment-Dispersion Ambiguity

- Moment-dispersion ambiguity sets

$$\mathcal{G} := \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{N_z}) \;\middle|\; \begin{array}{ll} \tilde{\boldsymbol{z}} \sim \mathbb{P} & \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] = \boldsymbol{\lambda} & \\ \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\boldsymbol{v}^\top(\tilde{\boldsymbol{z}} - \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right])\right)\right] \le \sum_{i=1}^{N_z} \lambda_i(e^{v_i} - v_i - 1) & \forall \boldsymbol{v} \in \mathbb{R}^{N_z} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \ge \boldsymbol{0}, \boldsymbol{A}\tilde{\boldsymbol{z}} \le \boldsymbol{b}\right] = 1 & \end{array} \right\}$$



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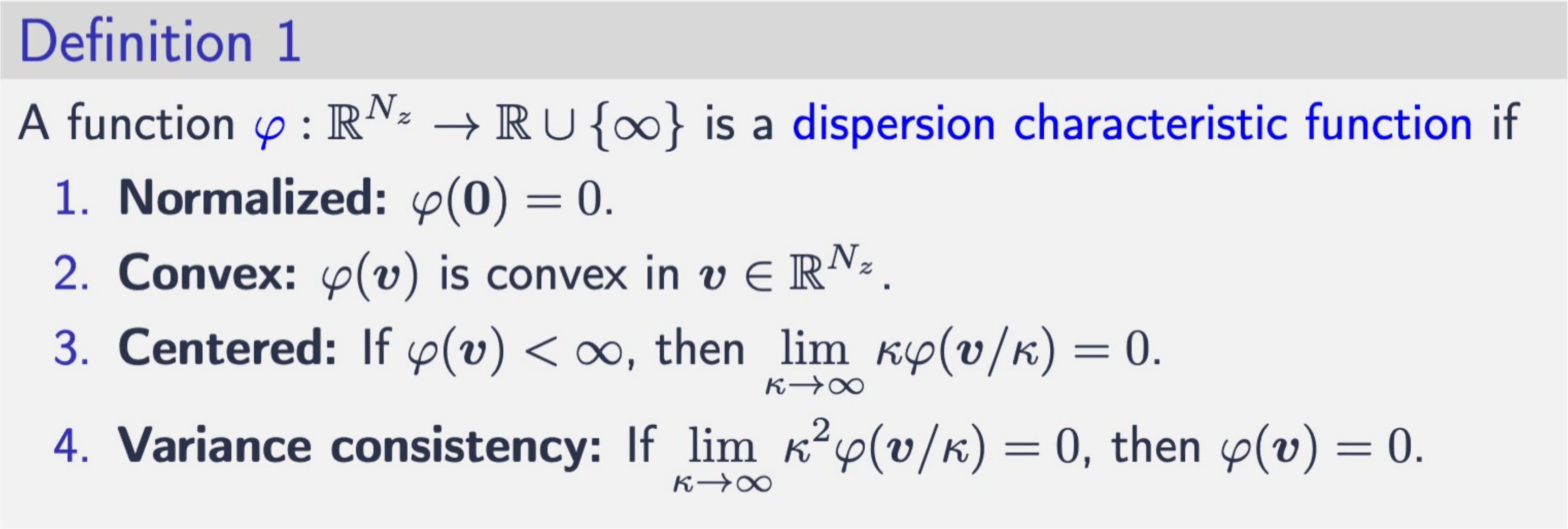
Moment-Dispersion Ambiguity

- Moment-dispersion ambiguity sets

$$\mathcal{G} := \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{N_z}) \;\middle|\; \begin{array}{l} \tilde{\boldsymbol{z}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] = \boldsymbol{\lambda} \\ \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\boldsymbol{v}^\top(\tilde{\boldsymbol{z}} - \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right])\right)\right] \leq \boxed{\frac{1}{2}\boldsymbol{v}^\top \boldsymbol{\Sigma} \boldsymbol{v}} \quad \forall \boldsymbol{v} \in \mathbb{R}^{N_z} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \geq \boldsymbol{0}, \boldsymbol{A}\tilde{\boldsymbol{z}} \leq \boldsymbol{b}\right] = 1 \end{array} \right\}$$



Definition 1

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Moment-Dispersion Ambiguity

- Moment-dispersion ambiguity sets

$$\mathcal{G} := \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{N_z}) \;\middle|\; \begin{array}{l} \tilde{\boldsymbol{z}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] \in \mathcal{W} \\ \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\boldsymbol{v}^\top(\tilde{\boldsymbol{z}} - \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right])\right)\right] \leq \varphi(\boldsymbol{v}) \quad \forall \boldsymbol{v} \in \mathbb{R}^{N_z} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \in \mathcal{Z}\right] = 1 \end{array} \right\}$$

- Distributionally robust optimization

$$\min_{\boldsymbol{x} \in \mathcal{X}} \sup_{\mathbb{P} \in \mathcal{G}} \mathbb{CE}_{\mathbb{P}}^{\kappa}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]$$

certainty equivalent
(entropic risk measure)

$$\mathbb{CE}_{\mathbb{P}}^{\kappa}\left[\tilde{\xi}\right] := \begin{cases} \text{ess sup}_{\mathbb{P}}[\tilde{\xi}] & \text{if } \kappa = 0 \\ \kappa \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\tilde{\xi}/\kappa\right)\right] & \text{if } \kappa > 0 \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\xi}\right] & \text{if } \kappa = \infty \end{cases}$$

General Framework

- Moment-dispersion ambiguity sets

$$\mathcal{G} := \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{N_z}) \;\middle|\; \begin{array}{l} \tilde{\boldsymbol{z}} \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right] \in \mathcal{W} \\ \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\boldsymbol{v}^\top(\tilde{\boldsymbol{z}} - \mathbb{E}_{\mathbb{P}}\left[\tilde{\boldsymbol{z}}\right])\right)\right] \le \varphi(\boldsymbol{v}) \quad \forall \boldsymbol{v} \in \mathbb{R}^{N_z} \\ \mathbb{P}\left[\tilde{\boldsymbol{z}} \in \mathcal{Z}\right] = 1 \end{array} \right\}$$

- Distributionally robust optimization

$$\min_{\boldsymbol{x}\in\mathcal{X}} \sup_{\mathbb{P}\in\mathcal{G}} \mathbb{CE}_{\mathbb{P}}^{\boldsymbol{\kappa}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]$$

risk tolerance level

$$\mathbb{CE}_{\mathbb{P}}^{\kappa}\left[\tilde{\xi}\right] := \begin{cases} \text{ess sup}_{\mathbb{P}}[\tilde{\xi}] & \text{if } \kappa = 0 \\ \kappa \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\tilde{\xi}/\kappa\right)\right] & \text{if } \kappa > 0 \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\xi}\right] & \text{if } \kappa = \infty \end{cases}$$

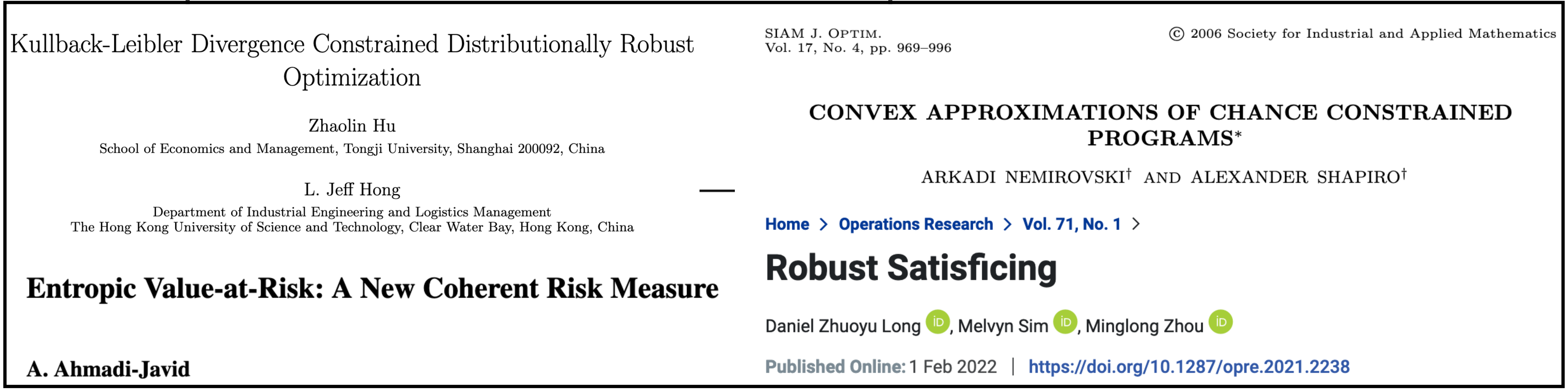
$$\mathbb{CE}^{\kappa}_{\mathbb{P}}\left[\tilde{\xi}\right] := \begin{cases} \text{ess sup}_{\mathbb{P}}[\tilde{\xi}] & \text{if } \kappa = 0 \\ \kappa \ln \mathbb{E}_{\mathbb{P}}\left[\exp\left(\tilde{\xi}/\kappa\right)\right] & \text{if } \kappa > 0 \\ \mathbb{E}_{\mathbb{P}}\left[\tilde{\xi}\right] & \text{if } \kappa = \infty \end{cases}$$

General Framework

- DRO under mean-dispersion ambiguity

$$\boxed{\min_{\boldsymbol{x}\in\mathcal{X}} \sup_{\mathbb{P}\in\mathcal{G}} \mathbb{CE}^{\kappa}_{\mathbb{P}}\left[f(\boldsymbol{x},\tilde{\boldsymbol{z}})\right]}$$

- Optimization models related to entropic risk measure



Kullback-Leibler Divergence Constrained Distributionally Robust Optimization

Zhaolin Hu
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Entropic Value-at-Risk: A New Coherent Risk Measure

A. Ahmadi-Javid

SIAM J. OPTIM.
Vol. 17, No. 4, pp. 969–996

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CONVEX APPROXIMATIONS OF CHANCE CONSTRAINED PROGRAMS*

ARKADI NEMIROVSKI† AND ALEXANDER SHAPIRO†

Home > Operations Research > Vol. 71, No. 1 >

Robust Satisficing

Daniel Zhuoyu Long, Melvyn Sim, Minglong Zhou

Published Online: 1 Feb 2022 | https://doi.org/10.1287/opre.2021.2238

Robust Optimization

- Tractable safe approximation

$$\boxed{\rho(\kappa, \boldsymbol{x}) \geq \sup_{\mathbb{P} \in \mathcal{G}} \mathbb{CE}_{\mathbb{P}}^{\kappa}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]}$$

  - conic representable: exponential cones, second-order cones, ...
  - non-increasing in κ to reflect decreasing risk aversion
  - good quality in risk-neutral case:

$$\boxed{\rho(\infty, \boldsymbol{x}) \leq \sup_{\mathbb{P} \in \mathcal{F}_L} \mathbb{E}_{\mathbb{P}}\left[f(\boldsymbol{x}, \tilde{\boldsymbol{z}})\right]}$$

  - not too conservative:

$$\boxed{\rho(\kappa, \boldsymbol{x}) \leq \sup_{\boldsymbol{z} \in \mathcal{Z}} f(\boldsymbol{x}, \boldsymbol{z})}$$

Exponential Conic Approximation

- Piecewise linear convex cost

$$\rho(\kappa,\boldsymbol{x}) \geq \sup_{\mathbb{P}\in\mathcal{G}} \mathbb{CE}_{\mathbb{P}}^{\kappa}\left[\max_{k\in[K]}\left\{\boldsymbol{a}_k(\boldsymbol{x})^\top\boldsymbol{z}+b_k(\boldsymbol{x})\right\}\right]$$

$$\begin{array}{rlll}
\rho(\kappa,\boldsymbol{x}) = & \inf & t_1+t_2+t_3 & \\
& \text{s.t.} & q_1+q_2=\kappa & \\
& & \displaystyle\sum_{k\in[K]}\eta_k \leq q_2 & \\
& & q_2\exp\left(\left(w_k+r_k-t_1\right)/q_2\right)\leq \eta_k & \forall k\in[K] \\
& & \boldsymbol{v}_k^\top\boldsymbol{\mu}+\bar{v}_k \leq w_k & \forall\boldsymbol{\mu}\in\mathcal{W}, k\in[K] \\
& & \left(r_k, q_2, \boldsymbol{v}_k-\boldsymbol{u}\right)\in\Phi & \forall k\in[K] \\
& & \left(\boldsymbol{a}_k(\boldsymbol{x})-\boldsymbol{v}_k\right)^\top\boldsymbol{z}+b_k(\boldsymbol{x})-\bar{v}_k\leq t_2 & \forall\boldsymbol{z}\in\mathcal{Z}, k\in[K] \\
& & \left(t_3, q_1, \boldsymbol{u}\right)\in\Phi & \\
& & \boldsymbol{t}\in\mathbb{R}^3, \boldsymbol{q}\in\mathbb{R}^2_+, \boldsymbol{u}\in\mathbb{R}^{N_z}, \boldsymbol{w},\boldsymbol{r},\boldsymbol{\eta}\in\mathbb{R}^K & \\
& & \boldsymbol{v}_k\in\mathbb{R}^{N_z}, \bar{v}_k\in\mathbb{R} & \forall k\in[K]
\end{array}$$

Exponential Conic Approximation

- Piecewise linear convex cost

$$\Phi = \mathrm{cl}\left\{(t, \kappa, \boldsymbol{v}) \in \mathbb{R}^2_+ \times \mathbb{R}^{N_z} \mid \kappa\varphi(\boldsymbol{v}/\kappa) \leq t\right\}$$

$$\begin{array}{rlll}
\rho(\kappa, \boldsymbol{x}) = & \inf & t_1 + t_2 + t_3 & \\
& \text{s.t.} & q_1 + q_2 = \kappa & \\
& & \displaystyle\sum_{k \in [K]} \eta_k \leq q_2 & \\
& & q_2 \exp\left(\left(w_k + r_k - t_1\right)/q_2\right) \leq \eta_k & \forall k \in [K] \\
& & \boldsymbol{v}_k^\top \boldsymbol{\mu} + \bar{v}_k \leq w_k & \forall \boldsymbol{\mu} \in \mathcal{W}, k \in [K] \\
& & (r_k, q_2, \boldsymbol{v}_k - \boldsymbol{u}) \in \Phi & \forall k \in [K] \\
& & (\boldsymbol{a}_k(\boldsymbol{x}) - \boldsymbol{v}_k)^\top \boldsymbol{z} + b_k(\boldsymbol{x}) - \bar{v}_k \leq t_2 & \forall \boldsymbol{z} \in \mathcal{Z}, k \in [K] \\
& & (t_3, q_1, \boldsymbol{u}) \in \Phi & \\
& & \boldsymbol{t} \in \mathbb{R}^3, \boldsymbol{q} \in \mathbb{R}^2_+, \boldsymbol{u} \in \mathbb{R}^{N_z}, \boldsymbol{w}, \boldsymbol{r}, \boldsymbol{\eta} \in \mathbb{R}^K & \\
& & \boldsymbol{v}_k \in \mathbb{R}^{N_z}, \bar{v}_k \in \mathbb{R} & \forall k \in [K]
\end{array}$$

Exponential Conic Approximation

- Piecewise linear convex cost
- Risk-neutral case

$$
\begin{array}{rlll}
\rho(\infty, \boldsymbol{x}) = & \inf & t_1 + t_2 & \\
& \text{s.t.} & \displaystyle\sum_{k \in [K]} \eta_k \leq q & \\
& & q \exp\left(\left(w_k + r_k - t_1\right) / q\right) \leq \eta_k & \forall k \in [K] \\
& & \boldsymbol{v}_k^\top \boldsymbol{\mu} + \bar{v}_k \leq w_k & \forall \boldsymbol{\mu} \in \mathcal{W}, k \in [K] \\
& & (r_k, q, \boldsymbol{v}_k - \boldsymbol{u}) \in \Phi & \forall k \in [K] \\
& & (\boldsymbol{a}_k(\boldsymbol{x}) - \boldsymbol{v}_k)^\top \boldsymbol{z} + b_k(\boldsymbol{x}) - \bar{v}_k \leq t_2 & \forall \boldsymbol{z} \in \mathcal{Z}, k \in [K] \\
& & \boldsymbol{t} \in \mathbb{R}^2, q \in \mathbb{R}_+, \boldsymbol{u} \in \mathbb{R}^{N_z}, \boldsymbol{w}, \boldsymbol{r}, \boldsymbol{\eta} \in \mathbb{R}^K & \\
& & \boldsymbol{v}_k \in \mathbb{R}^{N_z}, \bar{v}_k \in \mathbb{R} & \forall k \in [K]
\end{array}
$$

RSOME Implementation

```
class GMModel(ro.Model):

    def min_ce_infty_data(self, pieces, z, mu, sigmas, Gamma, fset):

        K = len(pieces)
        Nz = z.size

        t = self.dvar(2)
        q = self.dvar()
        u = self.dvar(Nz)
        w = self.dvar(K)
        r = self.dvar(K)
        eta = self.dvar(K)
        v = self.dvar((K, Nz))
        vbar = self.dvar(K)

        self.min(t.sum())

        self.st(eta.sum() <= q, q >= 0)
        self.st(rso.pexp(w + r - t[0], q) <= eta)
        self.st((v @ mu + vbar <= w).forall(fset))
        for k in range(K):
            self.in_Phi_Gamma_data(r[k], q, v[k] - u, sigmas, Gamma)
            self.st((pieces[k] - v[k] @ z - vbar[k] <= t[1]).forall(fset))
```

$$
\begin{array}{lll}
\inf & t_1 + t_2 & \\
\text{s.t.} & \displaystyle\sum_{k \in [K]} \eta_k \leq q & \\
& q \exp\left(\left(w_k + r_k - t_1 \right) / q \right) \leq \eta_k & \forall k \in [K] \\
& \boldsymbol{v}_k^\top \boldsymbol{\mu} + \bar{v}_k \leq w_k & \forall \boldsymbol{\mu} \in \mathcal{W}, k \in [K] \\
& (r_k, q, \boldsymbol{v}_k - \boldsymbol{u}) \in \Phi & \forall k \in [K] \\
& (\boldsymbol{a}_k(\boldsymbol{x}) - \boldsymbol{v}_k)^\top \boldsymbol{z} + b_k(\boldsymbol{x}) - \bar{v}_k \leq t_2 & \forall \boldsymbol{z} \in \mathcal{Z}, k \in [K] \\
& \boldsymbol{t} \in \mathbb{R}^2, q \in \mathbb{R}_+, \boldsymbol{u} \in \mathbb{R}^{N_z}, \boldsymbol{w}, \boldsymbol{r}, \boldsymbol{\eta} \in \mathbb{R}^K & \\
& \boldsymbol{v}_k \in \mathbb{R}^{N_z}, \bar{v}_k \in \mathbb{R} & \forall k \in [K]
\end{array}
$$

RSOME Implementation

```python
class GMModel(ro.Model):

    def min_ce_infty_data(self, pieces, z, mu, sigmas, Gamma, fset):

        K = len(pieces)
        Nz = z.size

        t = self.dvar(2)
        q = self.dvar()
        u = self.dvar(Nz)
        w = self.dvar(K)
        r = self.dvar(K)
        eta = self.dvar(K)
        v = self.dvar((K, Nz))
        vbar = self.dvar(K)

        self.min(t.sum())

        self.st(eta.sum() <= q, q >= 0)
        self.st(rso.pexp(w + r - t[0], q) <= eta)
        self.st((v @ mu + vbar <= w).forall(fset))
        for k in range(K):
            self.in_Phi_Gamma_data(r[k], q, v[k] - u, sigmas, Gamma)
            self.st((pieces[k] - v[k] @ z - vbar[k] <= t[1]).forall(fset))
```

$$
\begin{array}{lll}
\inf & t_1 + t_2 & \\
\text{s.t.} & \displaystyle\sum_{k \in [K]} \eta_k \leq q & \\
& q \exp\left(\left(w_k + r_k - t_1\right)/q\right) \leq \eta_k & \forall k \in [K] \\
& \boldsymbol{v}_k^\top \boldsymbol{\mu} + \bar{v}_k \leq w_k & \forall \boldsymbol{\mu} \in \mathcal{W}, k \in [K] \\
& (r_k, q, \boldsymbol{v}_k - \boldsymbol{u}) \in \Phi & \forall k \in [K] \\
& (\boldsymbol{a}_k(\boldsymbol{x}) - \boldsymbol{v}_k)^\top \boldsymbol{z} + b_k(\boldsymbol{x}) - \bar{v}_k \leq t_2 & \forall \boldsymbol{z} \in \mathcal{Z}, k \in [K] \\
& \boldsymbol{t} \in \mathbb{R}^2, q \in \mathbb{R}_+, \boldsymbol{u} \in \mathbb{R}^{N_z}, \boldsymbol{w}, \boldsymbol{r}, \boldsymbol{\eta} \in \mathbb{R}^K & \\
& \boldsymbol{v}_k \in \mathbb{R}^{N_z}, \bar{v}_k \in \mathbb{R} & \forall k \in [K]
\end{array}
$$

Summary

- Exponential conic optimization in RSOME + COPT
- Electric vehicle charging scheduling using exponential cones
- Robust optimization with moment-dispersion ambiguity

Thank you!

RSOME

COPT
Cardinal Optimizer